Modelling the Growth and Yield of Tropical Moist Forests

Jerome K Vanclay
Queensland Forest Service
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Preface

This book was originally intended to be a ‘‘How to do it’’ manual to guide foresters in the construction of growth models for the tropical moist forests. Unfortunately, modelling these forests isn’t that easy. There is no single ‘‘best’’ way to build a model for these forests. Rather, many approaches can be used, and the best one to use depends on the data available, the time and expertise available to build the model, the computing resources available, and the inferences that are to be drawn from the model. So rather than writing a ‘‘cookbook’’ with one or two recipes, I have chosen to give a detailed review, to illustrate to the reader the many approaches available, the requirements of and output from each, and their strengths and weaknesses. I also indicate what I believe to be the more promising approaches. Whilst this places more responsibility on the reader to choose and develop a suitable modelling methodology, it should also give the reader a better understanding and lead to better models and more reliable predictions. And I hope that better models will provide better information, greater understanding, and better management of the tropical moist forests.

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Jerry Vanclay
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1. Introduction

One way to ensure the continued survival of the tropical moist forest (TMF) is to manage it for commercial production of timber and other forest products, so that it becomes valuable in the eyes of local communities. Two conditions are essential, but not sufficient for its survival. Firstly to ensure that harvesting leaves the forest in an ecologically and silviculturally ''good'' condition. Secondly, to schedule the harvest to provide a continuing supply of timber and other benefits. Growth models, when combined with inventory, provide a reliable way to examine harvesting options, to determine the sustainable timber yield, and examine the impacts on other values of the forest.

The thrust of this book is not necessarily confined to the tropics or to moist forests. The book focuses on the special problems in modelling forests comprising many species, a wide range of trees sizes, and/or indeterminate ages. These problems are common to most TMF, and to many natural forests elsewhere.

1.1 What is a Growth Model?

The Oxford English Dictionary defines a model as the representation of some existing structure showing the proportions and arrangements of its component parts. The joint FAO/IUFRO committee (Ford-Robertson 1971) defined the word in the statistical sense as a formalized expression of a theory. In their recent textbook, Avery and Burkhart (1983:275) discussed tables, equations and models: ''Techniques for forecasting stand dynamics are collectively referred to as growth and yield models''. Common usage of the term generally refers to a system of equations which can predict the growth and yield of a forest stand under a wide variety of conditions. Thus a growth model may comprise a series of mathematical equations, the numerical values embedded in those equations, the logic necessary to link these equations in a meaningful way, and the computer code required to implement the model on a computer. Hagglund (1981) defined a growth simulator as ''a complete set of those prediction tools needed for forecasting growth in stands of one or several species, growing in a defined geographical region''.

Growth refers to the increase in volume of a stand over a given period of time (i.e. m^3/ha/ann). Yield refers to the total volume produced at the end of a certain period (i.e. m^3/ha). The difference between growth and yield models is obvious in even-aged stands, but less obvious in mixed stands. In even-aged stands, a growth equation predicts the growth of diameter, basal area or volume in units per annum from age and other stand characteristics, whilst a yield equation predicts the diameter, stand basal area or total volume production attained at a specified age. Thus a growth function says that at age t a stand is growing at dy/dt units per annum, whereas a yield function says that the stand of age t contains y units.

The simplest model is the plantation yield table, which may comprise only two columns of figures, the left column showing plantation age, and the right column indicating the expected standing volume at that age. This may also be expressed graphically as a single curve, with the horizontal axis indicating age and the vertical axis indicating volume produced, or more concisely as a mathematical equation. Several curves may be used to indicate expected yields for a range of site productivity. At the other extreme are complex models which may give the spatial positions of the individual trees, their diameter, height and crown size, may indicate timber quality and knot size, and may interface with conversion simulators which predict veneer and sawn timber outturn. Similarly, growth models for mixed forests may vary from the simple (e.g. Grimes and Pegg 1979) to the complex (e.g. Belcher *et al.* 1982, Wykoff *et al.* 1982).

A yield equation may be produced by integrating the growth equation, and conversely, a growth equation may be obtained by differentiating the yield equation. Yield may also be determined by projecting the growth model, one year at a time, through the required number of years and summing the increments. The latter is the more flexible approach, as the yield model does not allow for changes in the management regime. In uneven-aged forests, models which express the status of the forest at some future time are termed yield models, and those which express increment are growth models (Figure 1).

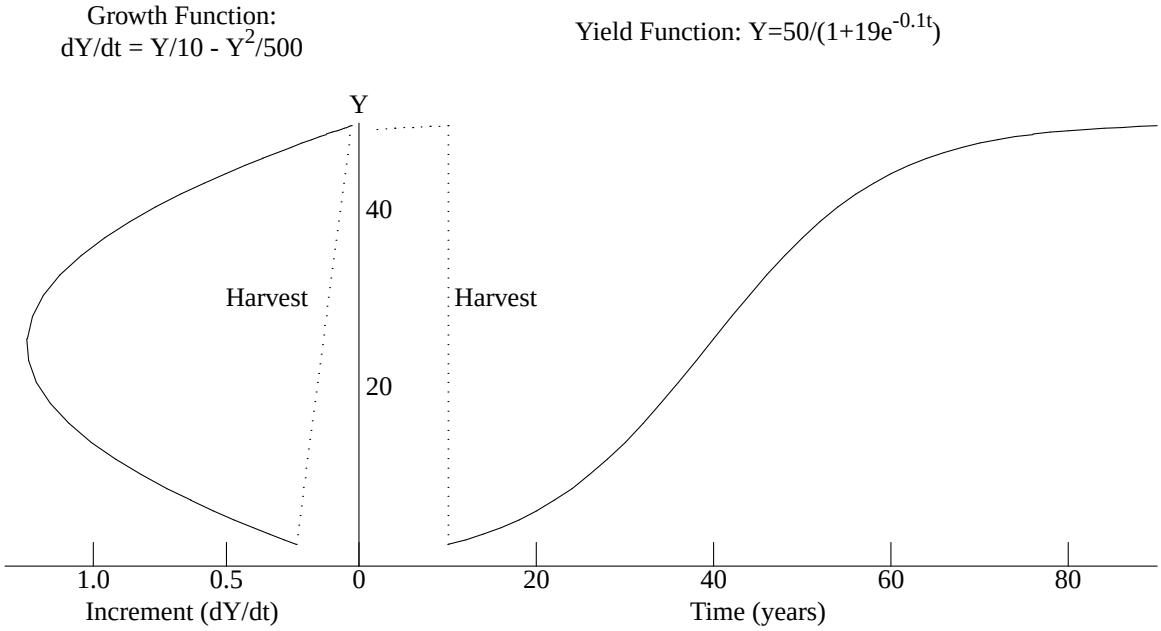

Figure 1. Growth and Yield Forms of the Logistic Function

1.2 Growth Modelling Approaches

There are so many growth models in existence that it is impossible to examine the methodology used in each, so it is necessary to identify some commonality, and to consider just a few examples for each class of model. Munro (1974) identified three philosophies: single tree distance-dependent, single tree distance-independent and whole stand distance-independent models. Ek and Monserud (1981) considered five approaches to modelling: traditional yield table methodology, differential or difference equations, stochastic processes, distributional methods, and individual tree simulation models.

An alternative classification could be based on the detail and the nature of the model. Depending on the amount of detail required, provided and utilized by the model, it may be considered a whole stand model, a stand class model, or an individual tree model. In addition, irrespective of the amount of detail, a model may be deterministic or stochastic.

Whole stand models are often simple and robust, but may involve complexities not possible in other approaches. Population parameters such as stocking (trees/hectare), stand basal area and standing volume are used to predict the growth of the forest, or the yield of the forest some years later. No details of the individual trees in the stand are determined. Stem size distributions are inferred from existing or predicted distributions.

Stand class models provide some information regarding the structure of the stand. A number of techniques are available to model stand structure, but one of the most widely used is the method of stand table projection which essentially produces a histogram of stem diameters at specified intervals. This approach is a compromise between whole stand models and individual tree models. When the class size is infinitely large and only one class exists, then the method is a whole stand approach. When the class width is infinitely small and each tree is considered a single class, then the method is the individual tree approach.

The most detailed approach is that of *individual tree models* which use the individual tree as the basic unit of modelling. The minimum input required is a list containing the size of every tree in the stand. Some models also require the spatial position of the tree, or tree height and crown class. Individual tree models may be very complex, modelling branches and internal stem characteristics, and may be linked to harvesting and conversion (i.e. sawing) simulators (e.g. Mitchell 1988, Vanclay 1988b).

TABLE 1. Examples of Modelling Approaches

Use	Resolution	Driving Variables	Example
EMPIRICAL MODELS			
Atmospheric Studies	Global primary production	Evapotranspiration	Lieth and Box (1972)
National Forest Planning	Stand variables	Age, stand basal area	Clutter (1963)
Regional Planning	Individual trees	Tree species & sizes	Prognosis (Stage 1973)
Silvicultural Studies	Tree crowns	Tree & branch variables	TASS (Mitchell 1975)
Silvicultural & Conversion Studies	Wood characters	Branches, ring width & density	SYLVER (Mitchell 1988) PLATIPUS (Vanclay 1988b)
HYBRID & PROCESS MODELS			
Ecological Studies	Individual trees	Tree species & sizes	Doyle (1981)
Nutrient Cycling	Individual trees	Trees, nutrients	FORCYTE (Kimmins 1988)
Physiological Studies	Mass of foliage, branches, roots	Biomass, photosynthesis, respiration	Sievanen *et al.* (1988)

Two other classes of models are beyond the scope of this book, but warrant a mention. Process models attempt to model the processes of growth, taking as input the light, temperature and soil nutrient levels, and modelling photosynthesis and the allocation of photosynthates to roots, stems and leaves (e.g. Landsberg 1986, West 1987, McMurtrie *et al.* 1990). These models help provide a better understanding of growth and stand dynamics, but have not yet successfully been used for predicting timber yields. Ecosystem succession models (West *et al.* 1981, Shugart 1984) attempt to model species succession, but are generally unable to provide reliable information on timber yields. FORCYTE (Kimmins 1988) is a hybrid approach and enables forest yields to be modelled under differing nutrient regimes.

A *deterministic* growth model produces an estimate of the expected growth of a forest stand, in the same manner as the mean indicates the expected trend for a population. Given the same initial conditions, a deterministic model will always predict the same result. However, because of natural variation in its environment, a real forest stand may not grow exactly the same amount each year, but may grow more or less than the expected amount.

A *stochastic* model attempts to illustrate this natural variation by producing different estimates of growth, each with a small probability of occurrence. Any one of these estimates may correspond exactly to the growth under some circumstances, but may differ from the expected growth. A single estimate of growth from a stochastic model is of little use, as a whole series of estimates is necessary to provide useful information of the variability of growth. For example, twenty estimates from a stochastic model will not only give a good indication of the expected growth (the mean or median), but also of its variability. The term ‘‘stochastic’’ is used in preference to the term ‘‘probabilistic’’, which is sometimes used to convey the same meaning, but also refers to stand table projection based methods which give a deterministic solution predicting fractions of trees rather than whole trees.

Deterministic and stochastic models serve complementary purposes. Deterministic models are effective for determining the expected yield, and may be useful in determining the optimum stand condition. Stochastic models indicate the the reliability of these predictions. Other methods such as variance propagation techniques (e.g. Gertner 1987a) may provide similar information more efficiently than stochastic models. Both deterministic and stochastic predictions can be obtained from some models. Although stochastic models can provide some useful information not available from deterministic models, most of the information needs for forest planners and managers can be provided more efficiently with deterministic models.

Figure 2. Components of Forest Growth

1.3 Components of a Model

The more detailed approaches to forest stand modelling need to identify components of forest growth to model the processes effectively. The nature of the components distinguished depends upon the forest type and the approach used. In models for intensively managed plantations, mortality and recruitment may frequently be ignored. However in TMF, these form an important aspect of the stand dynamics, and may have considerable influence on volume yield of the stand. In addition, the components identified in whole stand models tend to differ from those of individual tree models.

In individual tree models, the components usually identified are diameter increment, mortality and recruitment (Figure 2). Diameter increment is a simple concept and is relatively easy to predict. Prediction of mortality requires not only an estimate of the number of trees, but also the species and size of trees dying. Another aspect of change to be modelled is the deterioration of merchantable stems, which can be modelled in the same way as mortality. Recruitment may be predicted as ‘‘ingrowth’’ of trees reaching breast height, but some models may simulate trees from germination, others only after they reach some minimum size, 10 cm diameter for instance. Each of these components will be addressed individually in subsequent chapters.

1.4 Choosing a Model

The preceding discussion has hinted at a range of approaches used in modelling, and the following chapters will demonstrate the diversity of modelling techniques that have been used. It is appropriate at this stage to give a few guidelines for critical selection of an appropriate technique. A map provides a convenient analogy to growth models. To find your way with the least inconvenience, you must choose the map with the right amount of detail for your particular purpose. There is a place for national route planning maps, for topographic maps, and for urban street directories. The potential user should not have blind faith in the results produced by a computer, but should remember that ‘‘All models are wrong: some are more useful than others’’. The user should choose an appropriate model and avoid unnecessary complexity. ‘‘The aim of science is to seek the simplest explanation of complex facts ... Seek simplicity and distrust it’’ (Hunter 1982).

Clutter (1980) listed several fallacies that should NOT provide a basis for selecting a growth model:

1. Among the models existing at any point in time for a particular forest type, there is one, and only one, best model and all other models are inferior to it.
2. Complexity is desirable for its own sake.
3. Model quality is directly proportional to the amount of computer time required to run the model.
4. Nonlinear models are inherently superior to linear models.
5. Models written in APL are superior to ... models written in FORTRAN which are, in turn, superior to models written in BASIC.
6. Models which do not require a computer for their implementation are the most inferior of all and are unworthy of discussion.
7. Stochastic models are intellectually preferable to and aesthetically superior to deterministic models.

It is the end use that finally determines the best approach for modelling forest stands. Some applications for which growth models may be required include evaluation of site, testing adequacy of data, testing hypotheses of growth, estimating expected yield, estimating variability of yield, establishing the optimum management regime, discovering the effects of constraints on management, and evaluating the quality of the timber produced. Smith (1980) extended this list to 22 reasons why forest managers may require growth models, each of which may require a different approach to growth modelling.

Clutter (1980) suggested three guidelines for model selection which take into account the increasing intensity of forest management and the increasingly diverse markets for forest products. Models which provide a single measure of yield at a given point in time have little utility: they should indicate species and size class combinations. The unit of measure for yield should be flexible and should use cubic metres or weight, and not stacked cord or board foot volume. The model should be sufficiently flexible to accommodate a range of management regimes. Hyink (1990) argued that growth models should be continuous in the first derivative of key response variables (e.g. the diameter increment function should be smooth, not stepped), should be deterministic, and should extrapolate reasonably.

Buchman and Shifley (1983) provided a lengthy checklist to assess the suitability of a growth model for some stated application; key items include ease of use, accuracy of predictions and biological realism. Daniels and Burkhart (1988) stressed the need for reliable predictions, flexibility to simulate desired alternatives, sufficient detail for management decisions, and efficiency in providing this information. Models which are unnecessarily complicated may incur several costs: greater computational costs, loss in precision of estimates (Bruce and Wensel 1988, Sands 1988, Mowrer 1989), and difficulty in understanding and assessing the utility of the model. Complexity is no promise of reliability. Morton (1990) suggested that potential users should ask:

- Does the approach make sense?
- Will the model work for my application and input data?
- What range of data was used to develop the model and do these assumptions and inferences apply to my situation?
- What confidence can I place in model predictions? Users should be skeptical and demand proof.

Both simple and sophisticated models have their place. The appropriate approach depends upon the data, facilities and expertise available. What is important is what you *do* with the model. A sophisticated model is of no benefit if it is not used, whilst a very simple model may be of great benefit if it leads to better forest management. What is important is that the model is objective, unbiased, documented and available. Users

should understand how the model was derived, and should appreciate its strengths and weaknesses.

1.5 Examples

Many text books give a good overview of growth models for yield forecasting, and some good introductory texts include Avery and Burkhart (1983), Clutter *et al.* (1983) and Davis and Johnson (1987). Because this field is still developing rapidly, recent conference proceedings provide important reference material (e.g. Brown and Clarke 1980, Ek *et al.* 1988). Two widely used and well documented growth models are STEMS (Leary 1979, Belcher 1981, Belcher *et al.* 1982) and Prognosis (Stage 1973, Wykoff *et al.* 1982, Wykoff 1986).

Vanclay and Preston (1989) illustrated the use of a growth model for determining sustainable timber harvests from a TMF in Queensland. Their studies used a growth model developed from permanent sample plot data and a harvesting model developed from logging studies. The forest was stratified into management units, and temporary inventory plots were established in each unit (where no inventory data were available for a unit, plots were ''borrowed'' from sampled units considered similar). By repeatedly simulating the growth and harvesting of each individual plot and aggregating these results for each unit, future yields could be determined. Table 2 indicates characteristics of the future harvest, and illustrates its sustainability. Such analyses require sophisticated computer systems, but useful forecasts can be made with more simplistic approaches (e.g. Mendoza and Gumpal 1987, Preston and Vanclay 1988).

Growth models offer forest managers a powerful analytical tool to investigate quickly and efficiently, the response of the forest to various management regimes. They allow the manager to determine a regime which will maximize volume production, or which will maximize the production of a particular product. It also enables him to determine the effect of a revised logging programme to exploit a change in demand. He can investigate effects of any constraint on forest operations, and its effect on yield. But the most powerful feature is the ability of the model to assist the manager to accurately determine yields over long periods of time, so that he can make long term commitments to the capital intensive wood processing industry, secure in the knowledge that the forest will not be over-cut.

TABLE 2. Yield Scheduling Example (adapted from Vanclay and Preston 1989)

	Regional Average			Average Characteristics of Simulated Harvests				
Year	Basal Area	Merchantable Volume	Yield (m^3/ha)	ASV (m^3)	Size Distribution % <60:60-99:100+cm	Veneer Logs	Major Species Code† and %	
1990	40	24	18.9	2.9	10:75:14	30%	YWN 21	MSW 13
2037	38	20	18.0	2.8	10:87: 3	34%	QSA 22	MSW 14
2077	37	25	18.8	3.0	8:90: 2	40%	QSA 21	MSW 16
2119	36	29	17.5	3.0	8:90: 2	42%	QSA 20	MSW 17
2166	36	23	14.7	2.8	11:86: 3	43%	MSW 17	NSO 17
2201	37	18	13.4	2.7	13:84: 2	52%	MSW 21	NSO 16
2231	38	19	13.3	2.7	13:85: 1	57%	MSW 22	QMP 21
2253	39	21	14.4	2.8	13:86: 1	54%	QMP 22	MSW 19
2290	40	26	17.3	2.8	14:86: 1	56%	QMP 23	MSW 19

† MSW maple silkwood *Flindersia pimenteliana*
NSO northern silky oak *Cardwellia sublimis*
QMP Queensland maple *Flindersia brayleyana*
QSA silver ash *Flindersia bourjotiana*
YWN yellow walnut *Beilschmiedia bancroftii*

2. Data for Growth Models

The first step in constructing a growth model is to obtain suitable data. All too often, the modelling approach is dictated by limitations of the data. Data requirements of many modelling approaches are similar and allow a set of minimum procedures to be established. The procedures discussed here relate to the requirements for development of growth and yield models. Additional details may be necessary if plots are to serve other uses such as ecological studies.

Stem analyses do not provide reliable growth data for many tree species in the TMF, so data must be obtained from remeasurements on permanent sample plots (PSPs). Many anomalies may be found in the growth rings of tropical tree species. Some evergreen trees (e.g. *Swietenia* spp.) may form rings while deciduous trees (e.g. some *Ficus* spp.) may not. Some species (e.g. *Hevea braziliensis*) may form several growth rings each year, while other species (e.g. *Shorea robusta*) may form only one ring but not necessarily in the same month each year. Mariaux (1981) reviewed the possibilities and problems of stem analysis in tropical tree species.

Permanent plots can never be completely replaced by temporary plots even for species amenable to stem analysis, because only PSPs allow satisfactory statistical comparisons within and between plots to check the adequacy of models, and only PSPs can provide reliable and consistent data on mortality, crown dynamics and stand level variables (McQuillan 1984).

Data used for growth research must be of a higher quality than that used for point-in-time inventories. For example, a diameter measurement of 50 ± 0.5 cm may seem precise, but if a remeasure indicates 51 ± 0.5 cm, the growth estimate will be 1 ± 0.7 cm which is not very precise.

2.1 Differing Data Needs

Sample plots serve many purposes, but different procedures are required to efficiently satisfy various needs of resource managers. Some information needs and corresponding sample plot procedures include:

- *Resource Inventory* (‘‘What is the present nature and extent of the resource?’’): Typically a large number of plots (or point samples) will be required to achieve the desired precision. Precision can be gained by orienting plots across environmental gradients to maximize within plot variation and thus reduce between plot variance. Cost considerations usually dictate that temporary inventory plots (or point samples) are most efficient for resource inventory. Specialized techniques for timber cruising offer great efficiencies (e.g. 3-P sampling), but may not provide data suitable for input to yield forecasting systems.
- *Continuous Forest Inventory for yield control:* If yield regulation is by volume control (e.g. Leuschner 1990:42), it is important that permanent plots be representative and established in various forest types and stand conditions in proportion to their area (this system is often called Continuous Forest Inventory). As with resource inventory, precision is gained by minimizing between plot variance. Plots should be marked so that they can be relocated for remeasurement, but should remain inconspicuous so that they receive unbiased management.
- *Growth Modelling:* The development of growth models for TMF requires the remeasurement of PSPs. As will become apparent later in this book, the most reliable and flexible modelling techniques require data in which the individual trees are identified. This requires that all trees on the PSP are permanently tagged and uniquely numbered. Irrespective of the modelling approach, unique numbering and tagging of trees is the only sure way of detecting measurement errors. Growth modelling also requires homogeneous plots, and this means minimizing within plot variance: the ability of the PSPs to quantify the present resource is irrelevant. Thus the same plot series cannot be efficiently used for both resource inventory and growth model development. If the growth model is to be used to investigate silvicultural and management alternatives, the database must include experimental data with paired treatment and control plots, both with adequate isolation. In contrast to

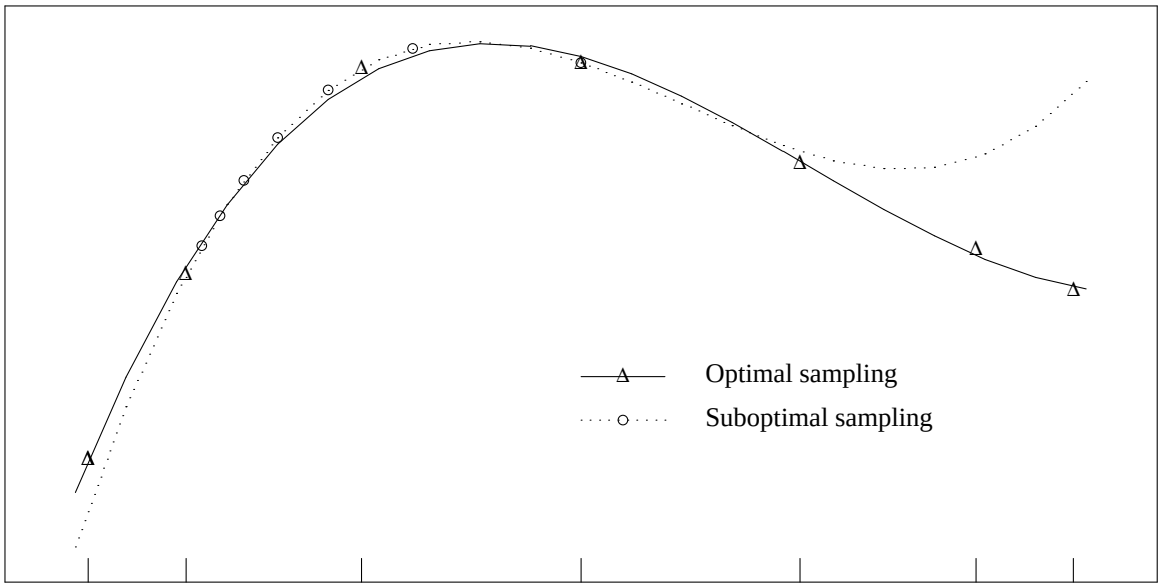

Figure 3. Polynomial approximation of a function with optimal and sub-optimal sampling

continuous forest inventory plots, it is not necessary for PSPs to be representative or numerically proportional to forest type areas, but it is essential that they sample the full range of stand conditions.

- *Long Term Monitoring of Environmental Change:* Dawkins and Field (1978) described a series of plots to monitor subtle long term changes in a forest. Whilst such studies are desirable, few organizations have the resources or need to establish such plots on the same scale as required for growth studies. Such detailed plots should be reserved for special studies. For growth modelling, it is better to sample the full range with conventional PSPs than to have a few detailed ''Dawkins'' plots. However, quantity is no substitute for quality.

Permanent sample plots established to provide data for growth modelling should be designed to satisfy this primary need, and should not be compromised in order to satisfy secondary needs. They need not provide efficient resource inventory data, as alternative sampling procedures can better fulfill that need.

2.1.1 Model Development

The initial and most obvious requirement for data is during model development when they are required for the construction of the basic functions comprising the model.

It can be shown mathematically, that the relationship between two variables given by a series of points known without error, can be described by polynomials most accurately if the sample points are located along the trend line with intensity increasing towards the limits of the region of interest. Figure 3 illustrates a cubic polynomial fitted to seven points selected from a diameter increment function (Vanclay 1991b, species group 1). The ticks on the horizontal axis indicate a near optimal sampling (Δ) which produced a very good approximation (solid line) to the original function. A more arbitrary sampling (o) resulted in a poor approximation (dotted line in Figure 3). In establishing a statistical relationship, these points are not known with certainty, but with some error, and the sampling intensity should reflect the variance. Extending this concept to a multi-dimensional space, it can be seen that sampling should be carried out across the entire response surface, with a greater intensity at the extremities, and in accordance with the variance. Computer programs are available to aid the optimal design of sampling schemes (e.g. Kennard

and Stone 1969). Limited but reliable data at each extreme and at the mean are more useful than copious data clustered about the mean.

Such data should not only sample a range of stand and trees conditions, but must also include remeasurements to enable detection of change, and must include a sufficient time period to average any climatic variations, and to ensure that growth patterns are not obscured by measurement error.

2.1.2 Validation and Monitoring

To provide a convincing demonstration, data used to validate a model should be excluded from its development and may comprise data drawn from a different population. This ideal is not always attainable, and it is common to partition the available data into two subsets, one to be used for development and the other for validation. It is important that the subset used for validation should contain at least some data collected over very long periods to allow detection of possible subtle but cumulative errors in the model. Where the model is used to estimate some optimum stand condition, it is advisable to obtain data to ascertain the production from this estimated optimum condition.

Monitoring may be viewed as continuing validation of a model by checking its operational predictions. It involves comparing projected and realised yields to identify any discrepancies. Such discrepancies may arise due to changes in management regime (especially logging practices), decline in site productivity, inaccurate resource data, or corruption of the validated growth model. Monitoring the performance of models is often neglected, but is necessary to ensure reliable forecasts.

2.1.3 Applications Data

Applications data may comprise basic resource information used in conjunction with the growth model to estimate future yields. Most operational resource inventory provides suitable data. Such data should detail areas of each homogeneous forest unit, the species composition, stand condition, and the site productivity of each unit.

Resource data have been integrated since inventories began, but recently attempts have been made to define principles and procedures for efficient integration (e.g. Lund 1986). Integration simply implies combining data conducted in different places, for different reasons, by different agencies or at different times. This may involve combining regional inventories to provide national or global statistics, finding correlations between timber inventory and soil or fauna survey data, or using growth models derived from PSP data to extrapolate data from temporary inventory plots to estimate sustainable yields.

Integrated inventory designs enable data from different inventories to be meaningfully combined. It does not necessarily mean that you have to measure everything in every inventory. On the contrary, it is better to do a few things well than to do a lot inadequately. However, in designing a PSP system, it is necessary to be aware of the information requirements of other researchers and other disciplines, and to consider how these requirements can be efficiently accommodated in PSP design. It is not necessary that all these requirements be satisfied immediately, but rather that the design accommodates these needs so that they can be phased in as required and when feasible.

2.2 Data for Modelling

2.2.1 Plot Selection and Establishment

Ideally, PSPs should sample the geographic range of the forest, and encompass a broad range of forest types, site productivity and topography. A broad range of stand basal area and tree sizes should be sampled for each tree species. Plots should include stands which have been subjected to a range of silvicultural management, including extremes of logging and treatment.

Growth of forests varies from year to year, fluctuations can be extreme, and mortality tends to be clustered in both time and space. Thus short time periods can give rise to biased growth estimates. Reliable growth models require PSPs with long measurement histories and adequate geographical distributions. Some of the plots should be left unlogged over long periods to ensure the most exacting validation.

Curtis and Hyink (1984) recommended that new PSP installations should be established only as part of a carefully planned series designed to give reasonable coverage of some defined range of site, geography, stand condition and treatment. The primary objective should be to provide data for defining response surfaces, thus studies should involve many locations with minimal replication at each location. Satisfactory growth models are dependent upon the availability of high-quality data from a wide range of stand conditions and treatments. Both ''passive monitoring'' data (i.e. survey data in undisturbed forest) and ''treatment response'' data (i.e. from paired treatment and control plots) from designed experiments are necessary.

Subjective location of PSPs may give rise to bias in the database if the environment cannot be completely quantified. It is preferable to locate PSPs randomly within a defined stratum of interest. This stratum may be defined on the basis of standing volume, species composition, soil type or any other objective means. Care needs to be taken when establishing plots at the forest edge or along roads and firebreaks, to avoid bias (Rennolls 1978, Fowler and Arvanitis 1979). Research plots may indicate higher yields than managed forests (Bruce 1977), and although some bias may be attributed to preferential management of visible plots, much of this bias arises from the subjective location of plots. The need for random location based on a thoughtful stratification cannot be over-emphasized.

There is some evidence that gains in precision can be achieved by sampling more large trees (e.g. Gertner 1987b), and it may be desirable to establish some plots around subjectively selected large trees. Such subjective selection of plots may introduce bias, but this may be an acceptable trade-off to reduce the variance associated with growth predictions from large trees. To minimize bias, these plots should constitute a small proportion of the total, and should be selected within strata based on site productivity and stand density (e.g. stand basal area).

Plots which are intended to be left unmanaged, for example to allow expression of density-dependent mortality and natural basal area in dense stands, should be clearly marked and excluded from any logging operations. Such plots which receive special management (no logging or more intensive treatment), should have adequate buffers to eliminate edge effects. The appropriate size of the buffer depends on tree size, but generally should be wider than the mature tree height. Other plots intended to receive routine management, should be marked in such a way as to be invisible to forest workers so as to ensure representative treatment. A plot should be ''difficult to recognize for those who do not know where it is, and easy to recognize for those who do and are looking for it'' (Synnott 1979). Plots must have unambiguous addresses and this requires precise grid references and detailed instructions for relocation of each plot. In some areas, some plots may suffer excessive trampling through high visitation. Dawkins and Field (1978) marked plot locations clearly, but 50 m away from the plot, and used invisible steel markers at all four plot corners.

2.2.2 Experiments

Although experimental data are extensively used in developing plantation growth models, it is more common to develop growth models for natural forests solely from passive monitoring data. In a designed experiment, all the variables are supposedly held constant except those varied according to the design; hence all the variables are accounted for. Data collected without the aid of an experimental design may be suspect for a variety of reasons. It is not unusual for both the explanatory and response variables to contain data errors. The explanatory variables are often highly correlated and frequently the region of interest is inadequately sampled. Another serious defect is that important variables may be omitted because their importance is unknown or the data are unavailable (Snee 1977).

Although logging and treatment in passive monitoring plots will influence stand density, other unknown factors may also determine stand density and composition. Thus, there is a very real danger that attempts to describe the behaviour of the stand as a function of stand density, for instance, will be confounded by the effects of site, pest and disease occurrence, and past history. To find out what happens to a system when you interfere with it, you have to interfere with it, not just passively observe it (Box 1966). Snedecor and Cochran (1980:356) discussed a study in which a survey revealed the unexpected result that the application

of farmyard manure reduced the yield of potatoes by half a tonne per hectare. In contrast, in controlled, randomized experiments, manure increased the yield by three to six tonnes per hectare. The difference may be due to the fact that those who had manure were livestock farmers with little interest in growing potatoes, and those who were most skillful at growing potatoes had no manure. Can we be sure that a similar problem in our PSP data is not troubling our attempts to develop growth models (e.g. stand density and site productivity interaction)?

Passive monitoring data may indicate greatest growth on the best sites with high standing basal areas, and little growth on poorer sites with little basal area. A growth model constructed from such data could suggest that greater increments accrue in stands with greater competition, since the effects of site quality and stand density could be confounded. Thus a model constructed from such passive monitoring data would predict a reduction in diameter increments following thinning, whilst a model from experimental data (e.g. thinning studies) would show an increase in diameter increment.

Consideration should be given to establishing a series of plots in homogeneous tracts of each forest type. Some should be left at maximum stocking to allow expression of density-dependent mortality and natural basal area, some should be logged and treated as a managed stand, and others should be heavily thinned to allow expression of open-grown development and regeneration. It does not matter that extreme treatments may never be applied in practice; they remain essential to properly define the response surface for growth models.

2.2.3 Number of Plots

The number of plots is usually determined by available resources. There is little point establishing more plots than can be maintained. It is better to have few plots providing reliable data, than many plots with inadequate management. The number of plots will also be determined by the variability of the forest estate, and the need to sample the full range of forest conditions. Synnott (1979) recommended 50 to 100 randomly located plots for each forest type.

A database comprising a few plots each with many remeasurements violates statistical assumptions of independence, and may require special analysis techniques (West *et al.* 1984, 1986). This violation becomes significant when the number of remeasures is large relative to the number of plots. An alternative is to use partial replacement, abandoning plots after several remeasures and establishing new ones (Tennent 1988). However, some plots must be retained for long periods with many remeasures to allow convincing validation.

2.2.4 Size and Shape of Plots

A general guide to the choice of plot shape is to minimize the plot edge to area ratio, and the number of corners. This leads to the choice of point samples, or circular, triangular, or four-sided plots according to the emphasis attached to corners and edges. Triangular plots are rarely used, perhaps because of the high edge to area ratio, and four-sided plots are generally rectangular (or square) to facilitate relocation of corners and boundaries.

Point samples (Beers and Miller 1964) have an advantage in being defined by a single point and a basal area factor, but they are inconvenient when dealing with recruitment, and create difficulties for the development of distance-dependent models. Circular plots are also defined by a single point and a radius, but the plot boundary becomes more difficult to define as the plot becomes large, as unlike polygonal plots, sight lines cannot be established along boundaries. As these plots are defined by a single marker (the centre), they may be more difficult to relocate if the marker is damaged or removed. Rectangular plots are more versatile. Plots marked by four corner pegs may be less likely to be lost than circular plots marked by only one peg. However, a more important reason for the choice of rectangular plots is their straight edges, few corners, and convenience. Square plots offer a theoretical advantage of minimum edge to area ratio.

Ideally, the plot size should be sufficiently small that the plot is homogeneous, at least with respect to forest type and site productivity, and sufficiently large to provide a representative sample of the forest stand. If a distance-dependent model is contemplated, the plot should be sufficiently large to allow estimates of competition to be determined for several trees on the plot. This leads to the conclusion that larger plots

offer greater flexibility, and the author's experience suggests that plots up to one hectare should be considered for growth modelling studies in TMF. Ecological studies may warrant even larger plots.

Curtis and Pope (1972) suggested that small plots may result in erratic estimates of stand attributes because of within stand clumping, and recommended the use of large plots. Payandeh (1974) examined the distribution to trees within north American forests, and reported that unlike plantations, natural forests rarely exhibit a regular spacing, but tend to have a random (in hardwoods) or slightly clustered (conifers) spatial distribution. Hann (1980) used plots varying in size from 0.3 to 0.5 hectares in even-aged stands, and 0.8 to 1.2 hectares in uneven-aged stands. Plots smaller than 0.8 hectares were available for the uneven-aged stands, but were discarded by Hann (1980) to avoid problems arising from within stand clumping. Synnott (1979) recommended square plots one hectare in area, subdivided into 25 subplots. West *et al.* (1988) found that 0.5 ha was the practical upper limit in TMF for homogeneous plots, as physical and floristic discontinuities hampered the establishment of larger plots.

Unless circular plots or point samples are adopted, the orientation of the plots needs to be considered. This may be inconsequential for square plots, but may be significant with elongated rectangular plots. Three possibilities exist. The plots may be randomly oriented, may be oriented according to the cardinal direction (e.g. long axis north-south), or may be oriented according to topography. In view of the need for plots which are homogeneous with regard to site productivity, the last of these is likely to be preferable. It is suggested that wherever possible, plots should be oriented with their long axis perpendicular to the slope (i.e. parallel to contours), or any other perceived gradient of site productivity to minimize within plot variation. In contrast, for temporary inventory plots for static resource estimation, it is desirable to maximize within plot variation so as to reduce the between plot variation and thus the sampling error. PSPs for growth model development have a different goal, and thus within plot variation should be minimized.

2.2.5 Measurement Procedures

Providing that PSPs continue to provide useful information, existing standards and procedures should be maintained to ensure uniformity. The continuity of standards is critical. However, when new plots are established or procedures for existing plots are revised, the following requirements should be accommodated (Whitmore 1989):

1. Divide each plot into subplots of maximum size 20 × 20 m. Mark corners permanently. Brightly coloured plastic pipe makes good rot-resistant pickets, but where elephants or primates are likely to destroy these, trenches may have to be dug instead. Since logging can completely destroy all plot marks, buried steel markers may be used in conjunction with these.

2. Trees must be numbered and permanently marked. Never use the same number twice. Do not re-use the number of a tree which dies but give ingrowth trees new numbers. Use an embossed aluminium tag fastened with an aluminium nail half driven home (to allow for growth). These nails need renewing about once every five years. Nails should be below the merchantable section, and should always be on the same side of the tree (e.g. northern or uphill) for ease of relocation.

3. Make a map to show the position of every tree to at the nearest 1 m or better. Work one subplot at a time. A maximum subplot size of 20 × 20 m makes mapping easy. Without a map confusions always occur because of death, ingrowth or lost number tags. Place measuring tapes along two adjacent sides of the subplot, and estimate and map coordinates of each tree while in the forest. Reed *et al.* (1989) suggested an alternative using pentaprisms.

4. Specify the minimum girth for the smallest tree to be included. This becomes important later when ingrowths occur. For the sake of clarity, note when surveys originally using imperial measure have been converted to metric measure. For example, 12 inches = 9.7 cm, but the metric minimum is commonly 10 cm.

5. Since girth can be measured sufficiently accurately only if always done at the same place, it is best to paint a ring on each tree with one straight edge. Painted rings need renewing every 3–4 yrs. In wet weather emulsion paint is easier to apply than oil-based paint. Some paints (especially oil-based paints) may cause abnormal bark shed and callus growth (e.g. *Eucalyptus maculata* and *Flindersia pimenteliana* in Queensland), so paints should be tested before general use. A less accurate alternative is always to measure girth a fixed distance above or below the number-tag nail. If a nail is used as the

reference point the girth should be measured a standard distance away from it (e.g. 20–50 cm) because some species develop swollen callus tissue around the nail itself. Make sure the girth measurement point is well above all buttresses, for these may grow upwards.

6. The measuring tape must be of metal or fibreglass because cloth tapes stretch when wet. All measuring tapes should be checked periodically against a reliable standard.
7. Since errors occur in recording girths, measure every tree twice independently. If the second reading differs, make a third one. Loose bark (except on species with corky or flaky bark), epiphytes and climbers must be removed from the line of measurement. The booker should always repeat the measurement for the measurer to confirm.
8. If, despite precautions at initial survey, a buttress grows up into the line of measurement, the line (and the reference nail if any) will need to be moved further up the bole although this is clearly not satisfactory, because it alters the girth measurement base for that tree. Measure at both the old and new heights, and record that a change in measurement height has occurred.
9. For trees with more than one trunk at the height of measurement decide the purpose of survey and, if necessary, create separate tree number and girth records. The usual convention is to treat such cases as two trees if forking is below breast height (1.3 m), and as one tree otherwise.

2.2.6 What to Measure

Plot location should be described and its coordinates (latitude and longitude or other grid coordinates of the SW corner, correct to the nearest second or better) and orientation (direction of long axis) should be recorded. Topographic features (altitude, slope, aspect, distance to ridge), climate (rainfall amount and distribution), indicator plants, and soil physical characteristics (depth, texture) should also be recorded. Uniformity of site should also be assessed. These variables need only be recorded at plot establishment, and at occasional remeasures.

Factors which may cause fluctuations in the observed growth include drought, heavy seed crops, pest populations, disease outbreaks and fire damage. Such information may assist the validation of models and guide the interpretation of outliers in regression analysis. Thus the occurrence of such events should be recorded on the plot measure record.

Synnott (1979) observed that it is impossible to measure all trees of all sizes and species, and is irrelevant to measure all seedlings and saplings since most will die. He recommended that all trees (of all species including useless stems) exceeding 10 cm d.b.h. (diameter breast high over bark) be measured. This is necessary to enable estimates of stand basal area, an important variable in predicting tree growth and stand dynamics. Limits as small as 3 cm have been adopted in natural coniferous forests (e.g. Arney 1985, Vanclay 1988a), but may be impractical in TMF. Reliable ingrowth data require that the measurement limit is less than the desired ingrowth size (e.g. for recruitment at 10 cm d.b.h., measure all trees exceeding 7 cm). Subplots within the main plot may be used to record data on stems smaller than this measurement limit.

Tree diameter and status (alive/dead/felled and cause) should be recorded at every measure. Every tree present at the previous measure should be accounted for. Height, crown parameters and estimated defect should be recorded at establishment, and periodically at remeasurements. Diameters should be measured at 1.3 metres (from the ground on the uphill side of the tree) or above buttress, and should be measured perpendicular to the axis of the tree (Carron 1968:17). The vantage point for height measurement should be carefully chosen to allow good visibility and a sighting angle of around 45 degrees (Romesburg and Mohai 1990). Natural mortality should be clearly distinguished from logging, treatment and other removals. Trees which appear lost should be marked as such, and should not be attributed to death unless there is evidence to support this.

The measure crew must check doubtful items and make sure that the current measurements are correct. They should record that such checks have been made. Decrements and other anomalies in the data should not be altered once the measure crew has departed the plot. Although these data may at times look unrealistic, ‘‘massaging’’ the database to alter these data may cause significant loss of information. At best, this practice may result in unrealistically low estimates of standard error associated with any functions developed. At worst, it may exclude the opportunity to investigate some originally unsuspected event (e.g.

weather patterns and climate change) or unforeseen topic. There may be good reasons to ‘‘massage’’ or omit data from some specific analysis, but the main database should never be altered.

The quality and cost of data available for analysis may be improved substantially through the use of electronic data recorders (Fins and Rust 1987, Leech *et al.* 1989, Wood 1990). Electronic hand-held devices enable basic checks of the input data to eliminate simple errors (e.g. transposition) at their source, instant validation, comparisons with previous measure, and speedy transfer of data to database. They can ensure that the measurer does not progress to the next tree or plot until all necessary variables have been recorded.

In summary, the following variables should be measured:

- At the initial enumeration (and occasionally remeasured as new technology improves the precision that can be attained):
  - — plot location, dimensions and area,
  - — tree species and coordinates,
  - — topography (altitude, aspect, slope, relative position on slope),
  - — forest type,
  - — floristics (all species on plot and relative abundance),
  - — physical soil characteristics (depth, texture, colour, parent material), and an
  - — indication of the uniformity of the site;
- At the first measure, immediately after logging, and periodically (e.g. every second or third measure):
  - — sufficient tree heights for the determination of site productivity (or data necessary for alternative estimates of site productivity),
  - — merchantable heights and defect assessments of all stems (including non-commercial species, as utilization standards may change with time),
  - — crown characteristics (dominance, size, density, position, etc.), and
  - — basal area counts at each tree (unnecessary if tree coordinates are recorded);
- At every measure, assess all stems (including non-commercial, every stem from previous measure must be reconciled) for:
  - — diameter (over bark, breast high or above buttress) and height to measure point,
  - — validity (to indicate defects at measure point and anomalous but correct increments),
  - — status (alive, dead, logged, treated), and
  - — tree coordinates (recruits only);
- As necessary, record the occurrence of:
  - — logging, treatment and other management activities, and the prescription used,
  - — scars and other damage which may affect measurements or growth,
  - — meteorological phenomena (drought, flood, etc.),
  - — mast years (heavy seed crops),
  - — pests, diseases, fire, or
  - — any other aspect which may affect growth.

2.2.7 When to Remeasure

Theoretically, the frequency with which plots should be remeasured is influenced by two factors: ease of re-locating and identifying trees, and the rate of growth or change relative to the measurement error.

Remeasurements must be sufficiently frequent to ensure that the location of the plot and/or identities of stems are not lost; in some forest types this may be as frequent as every two years. Conversely, cost efficiencies demand that remeasurements should not be unnecessarily frequent. The increment of stems should be substantially greater than the error associated with the measurement if the remeasure is to be useful; shorter intervals lead to excessive variance in regression functions. Arney’s (1985) model of temperate coniferous forest used only data from measure intervals of 4 years or more, and Vanclay’s (1991b) TMF model used data spanning approximately 5 years. In TMF, an interval of two to five years may be appropriate for the remeasurement of PSPs.

As annual increments are generally required, measurements should be taken on the anniversary of the previous measure whenever possible, especially for annual or biennial measurements. Always try to measure during the same season, as trees in the seasonal tropics may exhibit marked seasonal fluctuations in girth due to changes in xylem water tension (Lieberman 1982). Remeasurements should avoid periods of rapid change (e.g. bark shed, rapid growth), should aim to measure during dormancy where it occurs, and should try to replicate environmental conditions at the previous measure (e.g. avoid remeasuring immediately after rain if the previous measure was after a long dry spell). Remeasurements should also be taken at the time of (preferably both before and after) logging (or silvicultural treatment), and as soon as possible after wildfire, cyclone or other disturbance. Knowledge of impending logging or treatment is required, particularly if plot boundaries are concealed, so that the necessary measures can be arranged. It may be desirable to mark plots with buried steel pegs prior to logging to insure against the loss of plot markers.

2.2.8 Administration

Administrative and office procedures associated with maintaining PSP measurement records are often neglected and can be a major cause of loss of information. Aspects to consider include the design of field forms, copying and storing the completed forms, and transferring the data to computer.

Forms should be designed specifically for the purpose of PSP measurement, and every column should be clearly marked with the data to be recorded. The form should have no redundant fields, and staff should become accustomed to completing every field. Every form should indicate plot identity, date, page number (e.g. page 1 of 3) and the name of the assessing officer, and should be completed before departing the plot. Forms should be completed using a sharp dark pencil, and no alterations should be made after departing the plot. Forms should not be transcribed, as this invites transposition and other errors. Any duplicate copies required should be photocopied (where photocopiers are not available, transcriptions should be clearly marked as such, and should be carefully checked by a third person). Forms should be filed securely and unambiguously, preferably with one plot per folder, with the forms arranged in chronological order. It is a wise precaution to have a copy of the data stored in a remote location (e.g. district and head office).

Details from the previous measure should be available during plot remeasures. This can be achieved by printing remeasure forms by computer and including details of previous measurements. Alternatively, this can be achieved by photocopying parts of the previous measure onto the new measure sheet, or by downloading previous data onto a electronic data recorder.

As data entry detects many illegible characters, errors and omissions, data should be entered onto computer as soon as practicable after collection, while the measure crew still recall some details of the plot. Data should be verified (i.e. re-entered and compared) by a different operator to detect any errors in data entry. Electronic data recorders offer several advantages, including such validation at the point of data collection, where checks can can be made. Validation programs should check the data for further errors and omissions, and summary reports should be produced for the information of assessing officers and forest managers. Copies of the data should be made and stored in secure remote locations.

Obvious errors and omissions in the computer data file should be amended, but the temptation to ‘‘massage’’ the data so that it all looks consistent should be avoided. The data on the computer must accurately reflect the field measurements. An anomalous measurement may or may not be due to measurement error in the field, and the database administrator’s adjustment remains a guess rather than a fact. Any alteration inserted by the database administrator should be clearly indicated as such (in the ‘‘validity’’ field), and these alterations should be kept to a minimum. It is much safer to let users edit their own copy of the data as necessary for their own analyses, than to alter the master copy.

An effective system requires a considerable commitment in staff and resources to initiate and maintain the PSPs, and this commitment must be on-going. Sufficient resources and trained staff are essential, or the quality and utility of data will deteriorate.

2.2.9 Re-appraisal

Periodic reappraisal of data collection policy and practice is necessary to ensure that the data being

collected are fulfilling current and perceived future needs. ‘‘The quality of data is of extreme importance. Competent, well motivated and supervised field crews are needed for measurement, and the control of all research plots should be vested in a single high level authority’’ (Deadman 1979). Reappraisals should address specifically two questions concerning data quality: ‘‘Is the specified quality adequate for current and perceived future needs?’’ and ‘‘Is the specified quality being attained’’.

The need to sample extremes of forest condition has already been discussed. However, the concept of what is extreme changes over time. Thus it is necessary to consider if the extremes being sampled are sufficient, and if not, new plots should be established. The cost of data collection and handling is high, so plots should be abandoned when no longer useful. The decision to terminate a plot with a long measurement history should not be taken lightly, as these plots will be most valuable for validation. However, it is inevitable that natural (and human) perturbations (e.g. lightning strike, landslip, insect or fungal attack) will extensively modify some plots. Such plots may no longer provide useful tree growth data, but may provide useful regeneration and other ecological data.

Perception of future requirements will change over time, so the data collection policy should be periodically amended to conform with these perceived needs. These amendments may require the termination of some plots, establishment of others, addition of new variables to be measured, or the deletion of others. However, changes in measurement procedures (especially deletion of variables) should not be undertaken lightly; stable, consistent measurement procedures are essential for growth research. There should be substantial and continued resistance to changing the plot measurement system (McQuillan 1984).

Permanent sample plots should satisfy the data requirements for growth models ten and more years hence. In order to provide for this next generation of growth models, it is appropriate to critically appraise the utility of the present PSPs, and to establish new plots specifically directed at collecting data for such future growth models. Such a series of ‘‘elite’’ plots should sample the range of forest conditions (and include thinning studies), but should be established in limited numbers so that appropriate care and attention can be given to detail and accuracy. The emphasis with these plots must be quality, not quantity.

2.3 Problems with Existing Data

The greatest problem facing many agencies is that the data necessary for growth model development are not available. Plots may not have been established, may have been neglected or abandoned, and measure records may have been lost. As there is little that can be done to salvage such lost data, it is imperative that care and attention are devoted to existing PSPs and their measure records. Other problems which severely restrict the utility of data include unreliable measurements, changes to procedures, and mistaken or undetermined species identities.

Data for the development of growth models for TMF may derive from PSPs which were established for purposes other than growth modelling. Such plots may sample a restricted range of stand conditions, omitting very poor and exceptionally productive sites, and avoiding extremes of stocking. Thus these data may not provide an efficient means to estimate response surfaces by regression equations to predict the behaviour of the forest under various conditions. Records concerning the establishment of many PSPs are sketchy or unavailable, and the reasons for the placement of these plots are frequently not clear. Some plots may have been randomly or haphazardly located in defined strata, but others may have been subjectively located. Any departure from a stratified random approach in establishing these plots requires some soul-searching on the part of the modeller, in considering the possible effects of personal bias in choosing plot locations, particularly where site quality cannot be reliably quantified.

PSPs should receive representative management (logging, treatment, etc.), except for experiments which sample extreme stand conditions. This may be assured where plots are marked with subterranean or other invisible markers, but intentional or unintentional bias in logging, treatment and other management may become significant when the plot is visible. Such management bias may not be a problem where it is reflected in the stand structure (e.g. removal of trees), but the effects of differences in logging damage and climber cutting may be more insidious. Differential management should be reflected in stand structure, but tests of some PSPs established for 50 years in Queensland failed to detect differences between PSPs and temporary plots established adjacent to them.

Growth in TMF is often highly variable, and this variation may be attributed, at least in part, to factors such

as weather, seed years, pest populations, disease outbreaks, fire damage, etc. Such information may be useful for interpreting apparently anomalous data detected during analyses, but is infrequently recorded and rarely transferred to the database. Evaluation of site productivity is a major obstacle in predicting yields from TMF, and development of a method for reliable site evaluation, and acquisition of the necessary data should be a priority.

How serious are these deficiencies so often present in data available for growth modelling? It is impossible to predict what difficulties these and other deficiencies may introduce, until the data are actually used in earnest. No data set can be perfect, but many will be found to contain deficiencies that will frustrate future analyses. Although plot remeasurement may appear to the measure crew to be unrewarding, collection and management of PSP data is vital to the informed management of TMF.

2.4 Conclusion

PSPs provide the basis for growth modelling, yield prediction and sustained yield management, and the reliability of the data is crucial to these and many other aspects of forest management. To obtain reliable data, it is necessary to ensure

— consistent standards,
— sampling a wide range of stand and site conditions,
— both passive monitoring and experimental plots,
— trees are numbered, marked and mapped,
— remeasurement frequency enables plot relocation and growth greater than measurement error, and
— measurement records are unambiguous and secure.

3. Forest Site Evaluation

It is obvious that some sites support luxuriant forest whilst others are capable only of supporting ‘‘poor’’ forests. This difference may be due to soil (fertility, drainage, etc.), climate (temperature and rainfall patterns), topography (altitude, aspect, etc.) and other factors, and may be reflected in the species present. Meaningful growth and yield forecasts require some evaluation of these site differences. Estimates of site productivity may influence growth, mortality and recruitment predictions, and must be accurate as any bias will affect all modelling results.

The following definitions based on Ford-Robertson (1971) will are used throughout this book:

Site Index is the stand top (or predominant) height attained by a stand at a specified index age, and is often estimated for stands of other ages using a height-age curve.

Site Quality is commonly used to refer to a descriptive measure of site determined by subjective methods, often by visual assessment into a relative (i.e. good-poor) classification.

Site Class refers to a more objective classification into a number of classes.

Site Productivity is used here as a general term for the potential of a certain species and site to produce timber. Site index, site quality and site class are approximate measures of the true site productivity.

The accuracy of any attempt to model forest systems depends largely upon the precision with which the site can be classified into homogeneous units (Gertner and Dzialowy 1984, Smith and Burkhart 1984). There are few techniques amenable to assessment of site productivity in TMF. Most texts ignore mixed forests, others suggest that a stratification by forest type is usually the most efficient. However, some reliable techniques are emerging.

A good measure of site productivity should be (Vanclay and Henry 1988):

- reproducible and consistent over long periods of time;
- indicative of the site, and not unduly influenced by stand condition or management history;
- correlated with the site’s productive potential; and
- at least as good as any other productivity measures available.

3.1 Types of Site Classifications

Methods for assessing site productivity can be classified on the basis of methodology into predictive or descriptive, and qualitative or quantitative approaches. However, these distinctions are not clear-cut, as there is a range of methodology varying from one extreme to the other.

Descriptive systems are those which require several years of measurement, or measurements several years apart, in order to arrive at an estimate of site productivity.

Predictive systems are those which require measurement of some site or crop characters at a single point in time to estimate the site productivity.

Qualitative systems result only in relative classes, which may be labelled numerically (Classes I, II, etc.) or otherwise (e.g. poor, good).

Quantitative systems use a continuous variable, frequently height, as a measure of site, and the resulting measure of site may be expressed as a real number, or grouped into classes.

Qualitative classes require that border-line cases be resolved. Expansion of the system to recognize more classes (either to include new extremes or to encompass more classes) is difficult. Thus careful consideration must be given to the number of classes to be identified. Provided the difference in growth rate is significant, there is no advantage in having fewer classes than can be recognized reliably (Lewis *et*

al. 1976:28). Quantitative systems are generally flexible and infinitely expandable, and eliminate the need to resolve border-line cases, but may give an inflated impression of precision.

Predictive approaches require measurement of some character of the site or stand and modifying it to derive an estimate of site productivity. Commonly such methods involve determining height at a known age, and transforming it to estimate site index or expected volume production. Site characteristics may also be used to estimate site productivity, and geographic regions, geology, vegetation types and crop appearance have also been used. For forest management purposes, predictive systems which enable site productivity to be estimated after a single visit are preferable to descriptive systems which require the maintenance and remeasurement of permanent plots. However, descriptive systems may be necessary to enable the development and validation of more efficient predictive systems.

If some general growth or yield model is applied to data from a number of permanent sample plots with repeated measurements, the residuals will indicate the site productivity of the plots. Large positive residuals indicate a better-than-average site, small residuals indicate an average site, and negative residuals indicate a poorer-than-average site. Suitable residuals can be obtained simply by plotting basal area increment against stand basal area, or by regression analyses of individual tree increments with the plot as a qualitative variable.

In either case, measurements over a period of several years are required before site productivity can be estimated for use in the prediction of future yields from the site. If some correlation can be discovered between the residuals and some easily measured site or crop parameter, this system may be the first step toward a predictive system. However, if no such correlation can be found, regional averages of these descriptive estimates of site productivity can be used as an interim measure.

Leary (1985) discussed a classification based on methodology (direct or indirect) and viewpoint (phytocentric or geocentric):

View	Method	
	Direct	Indirect
Phytocentric	Volume of wood	Tree height
Geocentric	Soil moisture & nutrient status	Climate
	Photosynthetically active radiation	Land form
		Physiography
		Plant indicators

The phytocentric view assumes that total stand volume or phytomass production is the ultimate measure of a site's productivity, while the geocentric view asserts the dependence of site productivity upon the soil and climatic factors. Although direct methods are preferable, they are more difficult to quantify, and this has led to the proliferation of indirect methods. ''The status of indirect phytocentric methods is so inflated that some speak of direct and indirect methods, not of site productivity estimation, but of site index estimation. This appears to be an unhealthy situation; what began as an interim solution (site index) to a difficult problem (geocentric approach) should not now be called the solution to the original problem'' (Leary 1985).

3.2 Assessing Site from Crop Properties

Crop properties such as site index are widely used as measures of plantation site productivity as they are usually easily measured and directly related to the utilizable production from the site. The most commonly used measure of site productivity is site index, but this requires an even-aged stand of uniform development. Various authors (e.g. Duerr and Gevorkiantz 1938) have tried to apply similar techniques to mixed forests by identifying a main even-aged stand in the forest. Stage (1963) proposed a method which used height, age and the rate of early diameter growth to compensate for early suppression. Careful selection of subject trees may overcome some problems, but many difficulties remain (Monserud 1988). However, all these techniques are of little use where age cannot be determined.

3.2.1 Stand Appearance

The appearance of the stand, or stand type, may provide an indication of the site productivity. Lewis *et al.* (1976:29) reported that *P. radiata* plantations develop recognizable stand differences in general vigour and form, in crown density, in needle length and colour, in tightness and colour of bark, in green level and in degree of canopy formation at age of assessment, particularly prior to thinning. The South Australian site quality classes (7 classes) can be recognized by experienced assessors from these qualitative characteristics. The method is relatively unaffected by the stocking variation normally experienced in South Australian plantations.

Vanclay (1989a) reported the use of visual assessment to classify TMF into two site quality classes (good, poor). Subjective assessments were generally reliable, and could be validated using a scoring procedure based on soil, species present, bole height and standing volume. Regression analyses of stand basal area increment on stand basal area suggested that additional classes offered no advantage. This does not reflect on the range of site productivity in these forests, but rather on the assessors ability to classify sites reliably.

3.2.2 Natural Basal Area

Pienaar and Turnbull (1973) observed that even-aged stands with initial stocking above a certain lower limit, converge towards an identical stand basal area, determined by the capacity of the site. If the premise that undisturbed sites tend toward equilibrium is accepted, then the equilibrium or natural basal area may be assumed to be an expression of the site's productivity (Assman 1961:229). This assumption is implicit in many growth models (e.g. Botkin *et al.* 1972, Alder 1980). However, as stand basal areas may fluctuate over time (especially on small plots) even when undisturbed, the approach may be liable to error. In logged stands, remeasurements over long periods are needed to estimate the equilibrium basal area.

Havel (1980b) reported the use of natural basal area as an indicator of site productivity in Western Australia. In mixed stands, natural basal area may depend upon species composition. The natural basal area of any given site may be lower for light demanding and crown-shy species than for shade tolerant species. Thus natural basal area may depend on the successional status of the stand.

3.2.3 Stand Height

The height attained by some species at the cessation of height growth is, in theory, a good indicator of site productivity (Westveld 1933). Stand height may be used as an estimator of site productivity if there are trees present in the stand which are sufficiently large to reflect the maximum potential height that the nominated species is likely to attain on that site. The concept is analogous to a site index with a very large index age.

Ogawa (1969) found potential maximum stand height a useful indicator of site productivity in tropical forests, and also found it to be highly correlated with total stand biomass. Havel (1975, 1980b) used stand height to estimate site productivity in jarrah (*Eucalyptus marginata*) forest in Western Australia. The average total height of dominant and co-dominant trees remaining after logging has been used as an indicator of site productivity of dipterocarp forests in the Philippines (Canonizado 1978, Mendoza and Gumpal 1987).

One difficulty of using stand height or total tree height is that the tree tops may be difficult to see in TMF. In such cases, useful results may be obtained using height to crown break or merchantable height. Other problems include the presence of emergent trees (e.g. *Araucaria*), the removal of large trees through logging, and wind damage to tree tops. Where suitably large trees are not available, height-diameter curves can be used to estimate the asymptotic height. Ogawa (1969) predicted maximum stand height using the height-diameter equation

$$\frac{1}{H_{\max}} = \frac{1}{H} - \frac{\beta}{DBH}$$

where $H_{\max}$ is the maximum stand height, H and DBH are pairs of height and diameter measurements on individual trees, and β is a parameter is be estimated. This equation can be derived by fitting the equation

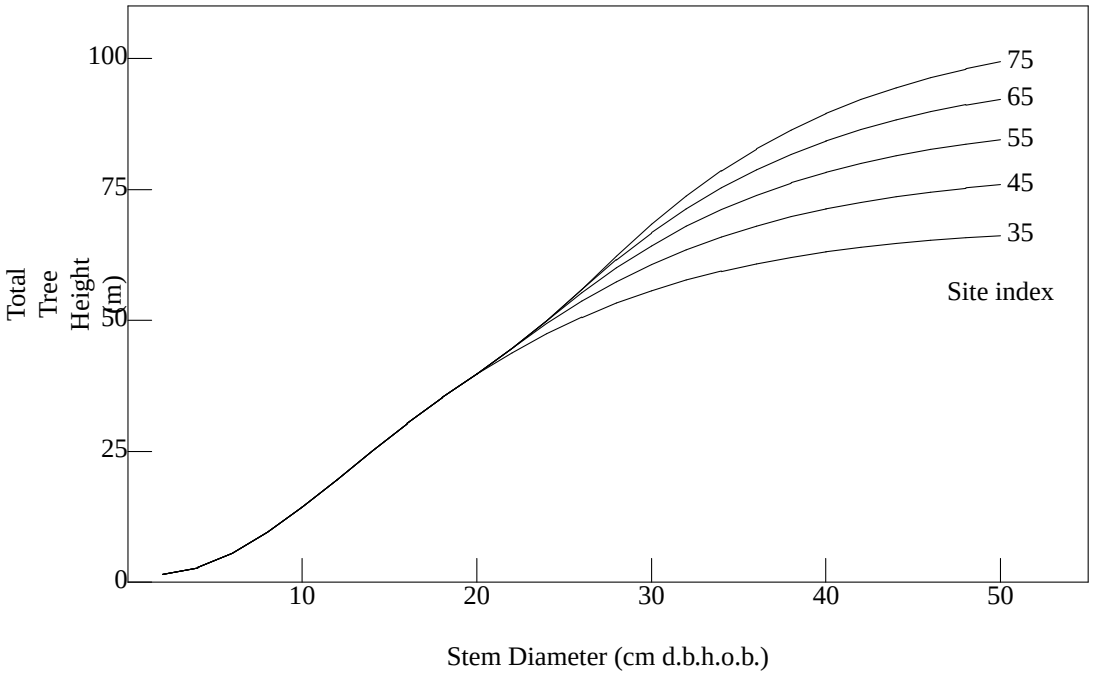

Figure 4. Site Index from the Height-Diameter Relationship (Reinhardt 1982)

$1/H = \alpha - \beta/DBH$ to several pairs of height and diameter measurements from individual trees, and estimating the maximum stand height as $H_{\max} = 1/\alpha$. However, extrapolation like this can be misleading, and interpolation is always preferable to extrapolation.

3.2.4 Height-Diameter Relationship

To avoid the need to extrapolate the height-diameter relationship, the height at a nominated index diameter can be used as a measure of site productivity; it has been suggested that this measure be called site form (Vanclay and Henry 1988) to avoid confusion with site index derived from the height-age relationship. The height-diameter relationship allows not only efficient evaluation of site in the field, but also the assessment of site from stereo aerial photographs (Reinhardt 1982) by estimation from crown widths and tree heights measured on the photographs.

McLintock and Bickford (1957) proposed anamorphic height-diameter equations based on dominant trees selected from stands from a wide range of sites, but not from stands with abnormal stocking or recent logging. Grimes and Pegg (1979) used hand-drawn height-diameter curves to characterize site productivity in spotted gum and ironbark forests in Queensland. Neither study attempted to relate these curves to site index, but used the expected height at an index diameter as a measure of site productivity.

Stout and Shumway (1982) and Lamson (1987) used height-diameter equations to predict site index compatible with published height-age equations. Their data were obtained from dominant and codominant trees, but taken only from even-aged stands. Reinhardt (1982) investigated several equations for the height-diameter-site relationship of western larch, and found that the relationship exhibited a strong polymorphic trend (Figure 4):

$$H = 1.3 + 8.23SI^{0.59} \times \left(1 - e^{-0.04DBH}\right)^{0.092SI}$$

where H is tree height (m), DBH is diameter (cm) and SI is site index (m) at 50 years.

Reinhardt (1982, 1983) worked with data from pure and mixed stands of western larch, and used the

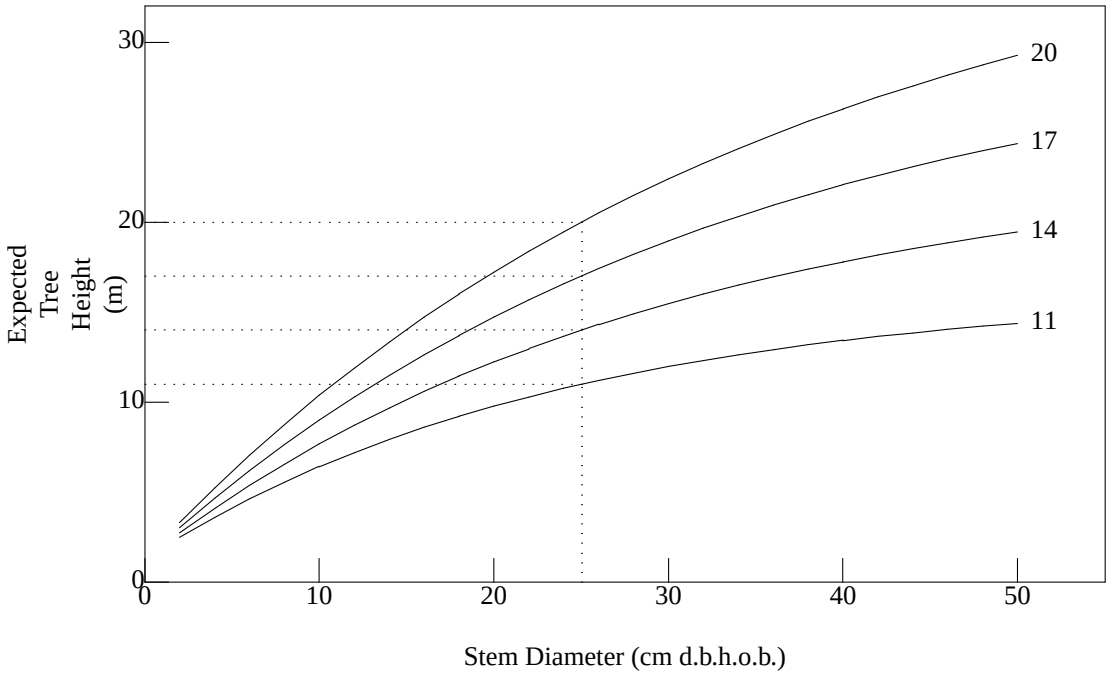

Figure 5. Height-Diameter Equations for *Callitris* (Vanclay and Henry 1988)

height-diameter curve to predict site index compatible with the height-age equations of Brickell (1970). The curves were not well differentiated for trees less than 40 cm d.b.h., and data from trees exceeding this diameter were necessary to establish a reliable relationship (Figure 4).

Vanclay and Henry (1988) adapted the method for use in the uneven-aged coniferous *Callitris* forests in Queensland, using the monomolecular or Mitscherlich equation (Figure 5):

$$H = A - (A - 1.3)\left(\frac{A - S}{A - 1.3}\right)^{\frac{DBH}{25}} \tag{3.1}$$

where H is tree height (m), A is the asymptotic stand height (m), $A = -10.87 + 2.46S$, and S is site form (m), the expected height of a 25 cm d.b.h. tree. An adequate estimate of site form could also be obtained from a simple linear regression of height on diameter ($H = \beta_0 + \beta_1 D$) for trees 20 to 30 cm d.b.h., and equation (3.1) was required only when trees of this size did not occur in the stand. Site form estimates for *Callitris* forests were relatively insensitive to logging, and remained constant over long periods of time. Logging may cause a perturbation in the estimate of site form for a few years, but the estimate stabilizes in a few years when the stand remains undisturbed. Site form is positively correlated with stand basal area increment and diameter increments of individual trees (Vanclay 1988a), and with several other indicators of site productivity (Table 3). Routine field application of this method suggests that best results are obtained in well-stocked monospecific stands.

The height-diameter relationship also showed promise as a measure of site productivity in mixed subtropical eucalypt forest in Queensland, and exhibited a strong correlation with volume production (e.g. 44 plots in *Eucalyptus* forest on Fraser Island in south east Queensland showed a correlation of 0.77 between site form and periodic annual volume increment). The use of several *Eucalyptus* and other Myrtaceous species to estimate site form on any plot did not appear to influence estimates.

Some important conclusions regarding the approach may be drawn by analogy from site index research. Heger (1973) and Curtis *et al.* (1974) demonstrated the influence of index age on the precision of site index estimation. However, the circumstances are somewhat different for the height-diameter relationship, as many natural forests may be expected to have a large range of stem diameters. The height-diameter relationship can be determined with least error if the index diameter is within the range of diameters

TABLE 3. Correlation coefficients between site form and site quality

Indicators of Site Productivity	Site Form (metres)	Site Quality (4 classes)
Subjective site quality estimate (4 classes)	.29	1.00
Maximum stand height (m)	.70	.40
Natural basal area (m^2/ha)	.46	.21
Periodic annual volume increment (m^3/ha/ann)	.40	.34

normally observed in the stand and if diameters are sufficiently large to allow height differences to be manifested.

Site index equations are commonly used for two purposes, to predict the height growth of a stand of known site index, and to estimate the site index of a stand of known age and height. Bruce (1925), Strand (1964) and Curtis *et al.* (1974) demonstrated the importance of using the correct form of equation in regression analysis. The response variable about which the errors are minimized, should be the variable of interest, in this case, height at the reference diameter. Some equations can be inverted and enable appropriate prediction functions for site form and for height to be formed. Parameters estimated for these alternative formulations usually differ (Monserud 1984, 1985).

Beck and Trousdell (1973) discussed sources of systematic error in determining site index equations, and their findings are applicable to the height-diameter relationship. The most common fault is to disregard possible age-site bias in sampling, and assume a constant curve shape on all sites. Beck and Trousdell (1973) stressed the need to ensure that the same sampling method used in establishing the equations, should be used in applying the equations.

3.2.5 Volume Production

Since volume production is usually the growth parameter of greatest interest to the forest manager, an evaluation of site productivity in terms of volume is desirable. The Association of German Forest Experiment Stations agreed in 1888 to site classification based on volume at 100 years. Sweden adopted a similar system in 1914, but since 1980 has estimated the mean annual volume increment at culmination (MAI). The Society of American Foresters (1923) recognized the superiority of a system based on MAI, but recommended that the use of site index based on height-age relationships was more convenient. The MAI cannot be measured directly for a single measure, but may be predicted from the height-age relationship.

The method of measuring volume must be standardized. Utilizable volume is inadequate because utilization standards vary in time and place. Estimates of sawn volume are even more unreliable. Assman (1961:75) recommended the use of solid wood (*derbholz*) volume defined as the volume under bark of all stem and branch material not less than 7 cm diameter under bark. This is convenient for conifers as it reflects the current utilization standards in common usage in many places. However, in trees with a deliquescent habit, this may entail the measurement of branch volume which may be difficult.

Mean annual increment at culmination is a concept which applies only to an even-aged stand, but the periodic annual increment (PAI) is analogous in TMF. Unfortunately, the complexities of TMF are such that even the PAI is of little utility. Even if the solid wood volume, or some other suitable standard could be determined reliably, the difficulties of predicting this PAI from some measurable crop parameter remain. Volume MAI is meaningful in monospecific industrial plantations, but the TMF contains many species which may vary greatly in productivity and wood density. Biomass production (tonnes/ha/ann dry weight) may provide a better basis for comparison. However, should the PAI refer to just one species, or to a specified mixture of species? In TMF, an index of the site is probably more useful than an index of a species on that site. Notwithstanding this, the historic volume production of a well stocked and well managed forest provides a good measure of site productivity, and a good benchmark to validate other more easily obtained measures of site productivity.

Direct measurement of volume PAI requires measurement of PSPs over many years. Schmoldt *et al.* (1985) attempted an alternative approach by fitting yield equations to PSP data, and examining both the maximum

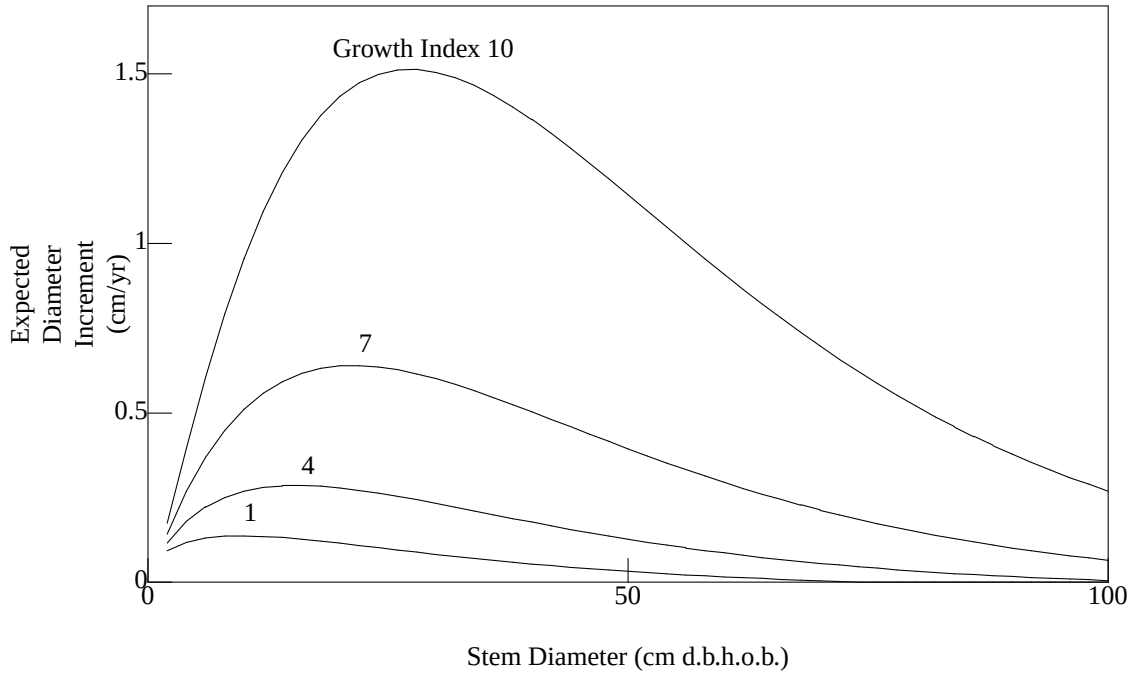

Figure 6. Growth Index equations for *Cardwellia sublimis* (Vanclay 1989c)
(Stand basal area=30 sq m/ha, overtopping basal area=10 sq m/ha)

growth rate and the asymptotic volume. They found that aspect and soil nutrients were significantly correlated with asymptotic basal area, maximum basal area increment and asymptotic volume, but not with maximum volume growth rate or site index. They suggested that site index is unreliable in mixed northern American hardwood forests, and suggested that coefficients from yield equations fitted to PSP data may provide practical alternatives.

3.2.6 Growth Index

Andel (1975) compiled a series of hand-drawn curves describing the basal area growth expected for a given stand basal area (*c.* 20 m^2/ha). His growth index indicated the expected basal area increment for a stand with the index basal area. Andel (1975) weighted the growth estimates to account for species composition of the stand in computing this index. Whilst this provides an indication of site productivity, it is liable to overestimate the site productivity for stands dominated by light demanding and pioneer species, and underestimate site productivity of stands dominated by slower growing and shade tolerant species.

Vanclay (1989c) developed an index of site productivity which accommodates a range of species and stand densities, does not require age or height, and avoids the problem with species composition in Andel's (1975) approach. The index was initially estimated from the historic measurement record for a number of permanent sample plots. Later, biotic and abiotic variables correlated with the index were used to predict the growth index for other sites and temporary plots. The index was based on the diameter increment of individual trees of several commonly occurring and widespread species. An increment function was fitted simultaneously to all species, with the plot identifier included as a qualitative variable (Figure 6). Basal area was included to account for differences in stand density. Site productivity was estimated from bioassay data:

$$GI = \frac{\sum_{ij} Log(DI_{ij} + \alpha) - \sum_{ij}\left(\beta_{0i} + \beta_{1i} D_{ij} + \beta_{2i} Log(D_{ij}) + \beta_{3i} Log(BA) + \beta_{4i} OBA_{ij}\right)}{0.08808 \times \sum_{ij} Log(D_{ij})} \quad (3.2)$$

where GI is the growth index of the plot, D_{ij} is the diameter (breast high or above buttress, over bark, in cm) of tree j of species i, DI_{ij} is its diameter increment ($cm\ y^{-1}$), OBA_{ij} is its ''overtopping basal area'', the basal area of trees within the plot that are bigger than tree ij ($m^2 ha^{-1}$), BA is the plot basal area ($m^2 ha^{-1}$), and the βs are parameters estimated by linear regression.

This equation estimates growth index, a measure of site productivity based on the diameter increment adjusted for tree size and competition, of all trees of eighteen reference species (*Acronychia acidula*, *Alphitonia whitei*, *Argyrodendron trifoliolatum*, *Cardwellia sublimis*, *Castanospora alphandii*, *Cryptocarya angulata*, *C. mackinnoniana*, *Darlingia darlingiana*, *Elaeocarpus largiflorens*, *Endiandra* sp. aff. *E. hypotephra*, *Flindersia bourjotiana*, *F. brayleyana*, *F. pimenteliana*, *Litsea leefeana*, *Sterculia laurifolia*, *Syzygium kuranda*, *Toechima erythrocarpum*, *Xanthophyllum octandrum*) using all available remeasures for the plot (except that where plots were remeasured more frequently, remeasurements were selected to achieve approximately 5 year intervals). The β's were estimated by fitting the following equation simultaneously for all these reference species in the development data set (80 plots, a further 64 plots were used for validation studies):

$$Log(DI_{ijk} + \alpha) = \sum_{i=1}^{18} \beta_{0i} Spp_i + \sum_{i=1}^{18} \beta_{1i} D_{ijk} Spp_i + \sum_{i=1}^{18} \beta_{2i} Log(D_{ijk}) Spp_i + \sum_{i=1}^{18} \beta_{3i} Log(BA_k) Spp_i + \sum_{i=1}^{18} \beta_{4i} OBA_{ijk} Spp_i + \sum_{k=1}^{80} \gamma_k Log(D_{ijk}) Plot_k$$

where DI_{ijk}, D_{ijk} and OBA_{ijk} are the diameter increment (cm/yr), initial diameter (cm) and overtopping basal area (m^2/ha) respectively for tree j of species i on plot k, BA_k is the stand basal area (m^2/ha) on plot k, Spp_i is a dummy (0,1) variable which takes the value 1 for trees of species i and zero otherwise, $Plot_k$ is a dummy variable which takes the value 1 for trees on plot k and zero otherwise, and α, β_i and γ_k are parameters to be estimated. This equation can be expressed more compactly using GLIM notation (Aitkin *et al.* 1989):

$$Log(DI + \alpha) = Spp + D.Spp + Log(D).Spp + Log(BA).Spp + OBA.Spp + Log(D).Plot$$

where *Spp* and *Plot* are qualitative variables. The parameter α was assigned the value 0.02 after inspection of residuals and examining the residual mean squares from a range of values. The value 0.08808 (in equation 3.2) was subjectively determined to scale the growth indices into the range 0–10. This procedure yielded a descriptive measure of site, in effect an objective ranking of permanent sample plots, and does not provide a predictive measure which can be applied to temporary plots. However, the growth index can be predicted from other measurable or observable biotic and abiotic factors.

3.3 Assessing Site from Physical Site Properties

Crop parameters such as site index require an established crop, and thus cannot be used where a suitable crop is not present. Alternative methods of site productivity appraisal are important for evaluating plantation viability, and the prediction site index from physical site properties has been researched thoroughly. The problem of assessing site productivity of bare land does not usually arise in TMF, but physical site properties may provide an alternative way to estimate site productivity. Measures such as growth index require re-measured plots, and it is desirable to have a method that provides an estimate of site productivity after a single visit. Some physical site properties can be measured easily during a single visit, and may provide an efficient way to estimate site productivity if a correlation can be established with growth index or other proven measures of site.

One problem with the use of non-crop approaches to site evaluation is that they are usually used to predict the estimated site productivity determined from some crop property using regression analysis. Thus the utility of physical site properties to predict site productivity cannot be assessed directly, but is usually compared with some crop property (e.g. site index), which unfortunately is all too often inaccurate. This problem is evident in Grey's (1979) attempt to relate site factors to site index. Grey considered three

measures of site productivity, Marsh's site index, Crowe's site index and Wessels' MAI, each of which was best predicted by a different subset of the site factors considered; no one factor was common to all three. This suggests that these three measures of site productivity were not sufficiently reliable to enable a consistent relationship to be established.

3.3.1 Climate

The best known climatic index of forest growth is Paterson's CVP index which was designed to predict the maximum growth potential in terms of volume production over large areas (Johnston *et al.* 1967:198). It is based on evapotranspiration, annual temperature range, mean annual precipitation, length of growing season and mean monthly temperature of the warmest month. Although it has been adopted on a national scale by a number of countries, it is probably only useful for economic geography and general forest statistics where estimates of potential production are required for large inaccessible and non-inventoried areas. Similar indices of net primary production for global atmospheric studies have been based on evapotranspiration (e.g. Lieth and Box 1972) and on temperature and precipitation (e.g. Esser 1984).

Czarnowski (1964) developed an equation with three climate parameters, three soil parameters and four species properties to predict the productive capacity of a species independently of age, anywhere on earth. Trials with his equation indicate that predictions were close to observed values for three species on four continents. The model was subsequently refined to predict site index (at age 20) of *P. radiata* as a function of seven chemical and two physical soil characteristics, and three climatic variables, and achieved a mean error of about 10 percent (Czarnowski *et al.* 1976). However, this equation lacks the general utility of Paterson's CVP index, and involves considerable effort in determining soil nutrient status.

Degree-days (number of days during which the mean temperature exceeds a specified temperature, usually 5 or 10 degrees C) and precipitation during growing season have been used to estimate site productivity of temperate forests (e.g. Farr and Harris 1979). The distribution of the rainfall is probably more important than the actual amount (Jackson and Gifford 1974, Jackson *et al.* 1975). In temperate regions, rainfall during the growing season is important, whilst in the seasonal tropics the dry season rainfall may be critical. These variables may be used directly, or predicted as a function of latitude and altitude.

3.3.2 Topography

Climatic variables can only give a general indication of site productivity because they fail to account for any local variations in site. An obvious refinement is to include topographic information relevant to the specific site. An advantage of using only climatic and topographic information is that these details can be readily gleaned from topographic maps (or air photos) and climatic records.

In areas of marked relief, topographic effects may be the dominant force controlling site productivity. Evans (1974) found that height at age twelve in *P. patula* plantations in Swaziland was highly correlated with altitude. Monserud (1988) found altitude and habitat type were the best predictors of site index in irregular Douglas-fir stands in north America. Site index of upland oaks in southeastern Ohio can be predicted with reasonable precision from aspect, slope shape and position on slope (Carmean 1967). Stage (1976) demonstrated the interacting effects of slope and aspect on the site index of white pine, and suggested a technique to locate the favoured aspects. However, these factors may be more relevant in temperate tropical forests. Vanclay (1989a) found that total annual rainfall, altitude, slope and aspect were not significantly correlated with the growth index of TMF in Queensland.

Most investigations into the relationship between topography and site productivity have used simple variables such as altitude, aspect and slope. Whilst these have produced some promising results in temperate forests, their predictive ability has not been demonstrated in the tropics or subtropics. In an analysis of vegetation types, Moore *et al.* (1990) found that ''steepness'' (the diversity of elevation within a 200 m region, indicative of erosional/depositional environment) and ''exposure'' (the average angle to the northern horizon, indicating shading by the topography) provided better predictions than slope and aspect. Vertical height above the nearest stream also provided good discriminations, better than the conventional position on slope (Moore, *pers. comm.*). This suggests that there remains scope for further research in this area.

3.3.3 Soils

The next logical step in refining the estimation of site productivity from physical characters is to include information about the soil. However, complications quickly arise. Soil depth, colour and texture are easily determined, but determination of nutrient concentrations requires laboratory analysis, and there is no satisfactory way of quantifying soil water supply on a single visit. Furthermore, soil nutrient concentration need not reflect the availability to forest trees, and may change seasonally.

Despite these difficulties, numerous studies involving soil analysis have been made. Carmean (1973) listed 793 publications dealing with forest soils in the north central states of the U.S.A., some of which report reliable estimates of site index from soil nutrient concentrations. Mader (1976) found site index and periodic annual volume increment of white pine strongly correlated with several topographic and soil physical and chemical variables. In contrast, site index was only weakly correlated with topographic and simple soil physical properties, and topographic variables alone were not significant. Fralish and Loucks (1975) found soil nutrient data only slightly increased the precision of site index estimates for aspen. Grey (1979) failed to find any significant correlation between *Pinus patula* site index and a number of soil morphological and chemical features. However, topographical features such as land surface class, slope, altitude and distance from ridge crest were significant. Schonau and Aldworth (1991) found that effective rooting depth was the most influential variable in predicting site index of black wattle plantations, but that phosphorus, potassium and organic carbon content of the topsoil were also significant.

Jackson and Gifford (1974) and Hunter and Gibson (1984) found that productivity (volume increment and site index respectively) of *P. radiata* plantations could be estimated from rainfall and soil chemical and physical characteristics. Truman *et al.* (1983) found that site index of *P. radiata* plantations could be predicted from foliar nutrients (phosphorus and calcium) as well as from soil nutrients, but both approaches require expensive sampling and laboratory analyses. The relationship between foliar nutrients and site productivity is likely to be species dependent. Mijers (1937) found no clear relationship between the chemical composition of forest soils in Indonesia and teak site quality classes, but found that site might be evaluated from soil physical conditions.

Wright and van Dyne (1971) studied 50 equations predicting site index from physical site factors for several species in different regions of the U.S.A, but found it difficult to generalize on the most important variable. On poorly drained sites, texture and depth to impermeable layers were crucial, on other sites topography, available water and soil depth (total depth, effective depth or depth of A horizon) were important.

Carmean (1979a) suggested that important soil features include surface soil depth, depth to mottling, depth to impermeable layer, effective soil depth, texture and stone content, structure, drainage and subsoil colour. Schmidt and Carmean (1988) reported that soil depth, stone and clay content, slope and subsoil pH were significant in predicting site index of *P. banksiana*.

Webb and Tracey (1967) found that site index of *Araucaria cunninghamii* could be predicted from surface geology and land form, within broad climatic zones. Soil mineral status was the dominant factor. Acid rock produces soils of low fertility while more basic parent material yields soils of high nutrient status. Slope, soil depth and texture and the presence of a clay subsoil with impeded soil drainage were also important factors. Webb (1969) reported that simple measurements of soil depth and aeration and total amounts of the major inorganic nutrients, could enable reasonably accurate estimates of site productivity to be made. The soil mineral status may be deduced from surface geology, or inferred from the structure of the forest.

Baker and Broadfoot (1977) devised a scheme of determining site index based on the contribution of a number of factors toward the height at index age. They used five physical soil properties, eight moisture parameters, six factors reflecting nutrient availability and four factors concerning aeration, all of which were easily assessed in the field. The method assumes that each factor is responsible for a certain percentage of tree growth, and that there is limited interaction between factors.

Turner *et al.* (1990) formulated a soil classification for intensive *P. radiata* plantations, designed to reveal potential nutritional deficiencies and other management limitations from physical soil parameters that did not require laboratory analysis. The system was based on characteristics that indicated innate nutrient supply, the development of roots, moisture supply, and other growth and management factors, and included parent rock, texture profile, depth to and nature of impeding layer, texture and condition of the uppermost

10 cm of soil, character of horizons and the condition and colour of the subsoil. Parent rock sets upper limits to the total amounts of clay, primary quartz and many plant nutrients that can be released through mineral weathering (Brewer 1954). Turvey *et al.* (1990) found that parent rock, and depth to and nature of impeding layer explained most of the variation in wood volume production. Parameter estimates from their equation could be summed to provide estimates of wood volume production at age 11 years. The Turner *et al.* (1990) soil classification system provided better estimates of productivity than did other more widely used classifications.

3.4 Assessing Site from Vegetative Characteristics

Many site properties such as available water and nutrient concentrations are not easily measured, so an alternative is to measure indicative variables such as composition of ground vegetation. No causal relationship is implied, but it is assumed that both ground vegetation and timber production are influenced by the same properties. Lowry (1976) studied many physical site characteristics only to conclude that the native vegetation appeared to be the best indicator of site productivity. Daubenmire (1976) concluded that vegetation reflects the sum of all the elements of the environment which are important to plants, and established six basic principles arguing for vegetation as the best method for assessing site productivity:

1. *Vegetation reflects the sum of all the elements of the environment which are important to plants.*
2. *The species with the highest competitive powers are the best indicators.*
3. *Forests consist of superimposed groups ("unions") which occur in different combinations over the landscape.*
4. *Each union is sensitive to certain special aspects of environment.*
5. *Many characters of vegetation have potential significance as ecologic indicators*
6. *Types of environment ("habitat types") are the most basic ecologic units of landscapes.*

Two main approaches can be distinguished: the classification approach (also known as subdivision, European or Braun-Blanquet) which uses the (potential) climax vegetation, and the ordination approach (also known as the Anglo-American or Clements) using indicator plants. Regions may also be stratified or ''classified'' on the basis of land form or other characteristics (e.g. Smalley 1986), and this procedure should not be confused with Braun-Blanquet classification of vegetation.

3.4.1 Classification

There are several variations on classification, but all use the potential climax vegetation to classify areas into a number of habitat or site types, which are considered to be effectively uniform in many respects (Havel 1980a). The classic example of floristic classification for site evaluation is Cajander's use of various associations of ground vegetation to predict site productivity in Finland (Rennie 1963). This approach has been used with only minor modifications in Europe and north America. Ure (1950) applied such a system in New Zealand, and was able to derive a satisfactory and rapid estimate of *P. radiata* site class from his habitat types. He cautioned that for the system to work, the vegetation must not have been burnt, ploughed or otherwise interfered with. The sides of roads should also be avoided. In a similar study, Daubenmire (1961) found that floristic classification was a useful and rapid method for predicting height growth and disease susceptibility of *P. ponderosa* in the U.S.A. Waenink (1974) found a slight correlation between forest floor vegetation (i.e. habitat type) and growth of Japanese Larch, which improved when watertable classes were also considered. He also found a strong correlation between forest floor vegetation and the history of phosphate fertilizer application.

Webb (1959) developed a classification scheme based on the physiognomic characteristics of Australian rainforests. Webb *et al.* (1970) subsequently proposed a more sophisticated approach based on physiognomic and structural characters. This approach was found to be as effective as the more common floristic approach to classification in indicating environmental conditions. One advantage of using

structural features is the ease and speed of data collection. The approach also appears to have greater applicability over diverse geographic regions. One shortcoming is that structural features are generally defined in an informal manner, and delineation of habitat types is not necessarily unique especially when prepared by workers not familiar with the approach (Webb *et al.* 1970).

Pfister and Arno (1980, Pfister *et al.* 1977) discussed possible applications of their habitat type classification for Montana. One danger in shifting emphasis from assessing forest site productivity to a system which fulfills broader multidisciplinary objectives is that in doing so the value of the classification for site assessment may be reduced. However, Monserud (1984) found classification into habitat types useful, enabling site index curves of different shapes to be prepared for each type. Classification may provide accurate estimates of site productivity when developed specifically for the purpose, but more general approaches that can be used for a variety of purposes may not predict the site productivity so well (Jones 1969).

One problem with classification is that it remains an imprecise science. Ecological and other non-mathematical classifications are unavoidably subjective, whilst the outcome of mathematical approaches depends largely on the algorithm chosen. Of the many algorithms available, only single linkage cluster analysis emphasizes the separation of clusters; other algorithms may maintain clusters without regard to the possibility that two similar units may be assigned to different major clusters (Gower 1967, Jardine and Sibson 1971, Sneath and Sokal 1973).

3.4.2 Ordination

Two approaches to ordination exist. The first and most widely used is to use the presence (and occasionally abundance) of certain plants as an indication of site productivity; the other uses physiognomic characters such as size and shape of leaves, and the height of the indicator plants. These methods are not mutually exclusive, and may be used in conjunction.

The presence and abundance of each plant indicator expresses a set of environmental conditions favourable to that species. A community of such plants may express much of the biologically relevant factors and interactions of the site. Thus the use of indicator plants comprising a community may express the integration of factors more flexibly than the climax vegetation approach. Webb *et al.* (1967, 1971) found that classification failed to give a sensitive indication of site productivity and that ordination could reflect environmental factors with must greater precision. Havel (1975) found ordination more appropriate than classification, and obtained the best results when all very rare and very common species were excluded from the ordination analysis.

Hodgkins (1960) and Corns and Pluth (1984) gave regression equations incorporating the presence and abundance of several plant species to predict site index of natural conifer stands. However, Griffin (1967) reported that the abundance of indicator plants is likely to be influenced by disturbance, whilst presence or absence is more stable.

MacLean and Bolsinger (1973a) and Wiant *et al.* (1975) gave equations to predict site index from the presence and absence of several indicator plants. Most of the indicator plants could be found even after drastic disturbance. In a similar study, Dyrness (1973) found indicator plants remarkably persistent, even after clearfelling and burning. Webb *et al.* (1971) found that site productivity could be determined from suitable indicator plants even after clearing and several years of management. Webb and Tracey (1967) also gave lists of pioneer species which indicate good and poor sites for hoop pine plantations. This is consistent with Daubenmire and Daubenmire's (1968) observations that the presence of ground flora was largely independent of the overstorey. However, indicator species need to be chosen carefully, as even apparently stable rainforest may have a relatively high species turnover rate (Poore 1968). Swaine *et al.* (1987) reported a species turnover of around one percent per year in undisturbed TMF. However, indicator species in TMF may not be greatly influenced by disturbance as Stocker (1981) reported that 82 species returned within 2 years after felling and burning of a TMF. Webb *et al.* (1967) and West *et al.* (1988) reported that both soil moisture and fertility affected species occurrence.

Carleton *et al.* (1985) examined the influence of temporal factors such as stand density and succession on understorey vegetation in northern Canada, and found that these have minimal influence on vegetation. They found that the understorey vegetation was most influenced by soil, and concluded that the understorey vegetation should provide a reliable indicator of site productivity. However, Schonau (1987) considered the

use of plant indicators more useful in temperate regions where there are fewer species, and concluded that vegetation on its own is generally not suitable to estimate site productivity satisfactorily. Vanclay (1989a) found that the growth index of TMF could be predicted from the presence of several tree species. If the correct taxonomy of indicator trees was known, geology contributed no further improvement. However, exact identification of rainforest trees is often difficult, and a single common name may refer to more than one species. Good estimates of growth index could be obtained if geology was used in conjunction with common names:

$$GI = \begin{pmatrix} 4.528 \times AL \\ 5.934 \times BV \\ 5.164 \times AV \\ 6.174 \times CG \\ 4.980 \times SM \\ 3.837 \times TG \end{pmatrix} + 1.144 \times BLO + 1.286 \times SBN - 1.020 \times VTX - 0.673 \times RAP + 1.027 \times BUA + 1.008 \times RBN - 1.223 \times CLL + 1.516 \times BGR$$

where all variables are dummy (0,1) variables which take the value one if the geology or species is present on the plot, and zero otherwise, and *BLO* is blush silky oak (*Bleasdalea bleasdalei* and *Opisthiolepis heterophylla*), *SBN* is salmon bean (*Archidendron vaillantii*), *VTX* is vitex (*Vitex acuminata*), *RAP* is rapanea (*Rapanea achradifolia*), *BUA* is buff alder (*Apodytes brachystylis*), *RBN* is rose butternut (*Blepharocarya involucrigera*), *CLL* is cinnamon laurel (*Cryptocarya cinnamomifolia* and some affiliated species), and *BGR* is brown gardenia (*Randia fitzalanii*), and where the geology *AL* is alluvial, *BV* is basic volcanic, *AV* is acid volcanic, *CG* is coarse granite, *SM* is sedimentary-metamorphic, and *TG* is fine-grained (‘‘Tully’’) granite. Note that whilst the various geology types are mutually exclusive, any number of species may be present and used to evaluate the growth index. None of these species are short-lived pioneer species, and the presence or absence of these species should be relatively independent of successional status and disturbance.

Keenan and Candy (1983) used the principal components of a presence/absence (1,0) matrix in which 1 indicated that the species (10 species or groups) occurred over at least 30% of the plot area (0.01 ha) covered by non-eucalypt vegetation. The 10 species groups were derived by omitting species which occurred on fewer than three of the 52 plots, and amalgamating some less frequent genera. Matrices comprising presence/absence (any occurrence of 28 species) and percentage cover data provided similar but inferior results. Principal components offer some advantages for investigation: they provide more than one linear combination of the (0,1) data and can thus reflect more than one environmental gradient, and component correlations of the principal components are unchanged by adding or subtracting other explanatory variables to the model (Keenan and Candy 1983). However, one disadvantage is that principal components are specific to each set of data: additional data may give rise to different principal components, and thus subsequent studies and predictions must use the original component correlations. Keenan and Candy (1983) found that the first principal component explained 29% of the total variation, more than the non-floristic site factors (slope, altitude, exposure, soil parent material, soil pH, soil drainage) which, although significant ($P<0.01$) collectively explained less than 9% of the total variation. principal components 7, 8 and 10 were also significant. Keenan and Candy’s (1983) analysis suggested that plant species were better predictors of height growth than human appraisals of non-floristic factors.

3.5 Practical Considerations

3.5.1 Mapping Site Productivity using Remote Sensing

Forest type mapping has been prepared from remote sensing for many years, and broad site productivity classes can often be defined. Aerial photographs may allow objective measurement of some measures of site productivity. Bonnor and Morrier (1981) used aerial photography to classify temperate mixed forest in Canada into 5 metre site index classes with 76% success, the remaining instances underestimated by one class. Goodwin (1988) reported the use of aerial photography to determine mature stand height in temperate *Eucalyptus* forest in Tasmania. Aldred and Bonnor (1985) suggested that airborne lasers may be useful for mapping stand height. However, all these methods require the ground to be sighted through the

canopy, and may be impractical in TMF.

Digital remote sensing from satellite (e.g. Landsat) offers potential for objective algorithm-based site productivity mapping. Fox *et al.* (1985) reported that broad site productivity classes could be estimated from aspect derived from digital topographic data combined with vegetation classes obtained through supervised classification of Landsat data. Vanclay (1989c) and Vanclay and Preston (1990) reported that growth index could be estimated directly from Landsat TM data and geology, although prediction equations may need to be re-calibrated for each Landsat scene. Vanclay and Preston (1990) found that the ratio of band 4 (near-infrared) and band 5 (mid-infrared) provided reasonable estimates of growth index, especially when used in conjunction with geological data:

$$GI = \begin{pmatrix} 15.75 \times AV \\ 18.13 \times BV \\ 18.42 \times CG \\ 17.59 \times SM \end{pmatrix} + 3.017 \times \frac{TM_4}{TM_5} - 0.284 \times TM_1$$

where AV, BV, CG, SM are dummy (0,1) variables representing Acid Volcanic, Basic Volcanic, Coarse grained Granite and Sedimentary-Metamorphic geology respectively, and where TM_1, TM_4 and TM_5 are the blue, near-infrared, and mid-infrared Landsat TM bands respectively. However, this equation should be re-calibrated for each image, and could not be extrapolated to other passes.

3.5.2 Mapping Site Productivity using Geographic Information Systems

Geographic information systems have become an important and useful tool in forest management, and offer some potential for mapping site productivity. Several studies (e.g. Austin *et al.* 1984, Moore *et al.* 1990) have demonstrated the utility of these systems for mapping vegetation types, and these studies suggest potential for mapping site productivity.

Turvey *et al.* (1990) found that soil parent material and soil depth were the primary determinants of *Pinus radiata* site productivity. Suitable soil parent material classes can be derived from published geological maps, whilst soil depth may be inferred from a digital elevation model. Moore *et al.* (1990) found that steepness (elevation diversity) was easily calculated with a digital elevation model and indicated erosional and depositional areas which may serve as a suitable proxy for soil depth.

3.5.3 Multiple Estimates

Different methods of site productivity assessment may give rise to differing estimates, and the forest manager may have no basis for resolving these differences. Choosing the most popular or well established technique is one alternative, others may include choosing the method which intuitively seems right, or taking the mean or median of all available estimates. If PSP data are available, the alternative techniques can be tested using standard procedures (e.g. Freese 1960, Reynolds 1984), but where no such data are available, the ‘‘true’’ value cannot be determined and alternative selection procedures are required. Reed and Jones (1989) suggested an objective approach based on the multitrait-multitemporal approach of the psychometric literature (Campbell and Fiske 1959) to reconcile different estimates of forest productivity. The method involves conceptualizing the relationships among the estimates, standardizing these estimates, calculating correlations and identifying correspondences between the approaches. The most consistent approach is assumed to be the most reliable.

3.5.4 Changing Species of Estimation

Where a crop parameter is used as a measure of site production, it may refer to a specific species in the crop, or to a stand of specified composition. It may be possible to gauge the potential of a site for another species or composition by a transformation of the measured parameter. Another application of this technique is that it enables extensive areas of mixed forest to be evaluated in terms of one standard species, even if that species is not present over the whole area.

Foster (1959) found a curvilinear relationship between the site indices of eastern white pine and red maple, and found that red maple exhibited rapid early height growth and could be expected to exceed the height of white pine until the age of 46 years, when the trend reversed. Red maple was more sensitive to site, being taller on good sites and shorter on poor sites, than white pine. McQuilkin (1974) found that site indices of black, scarlet and white oaks differed by simple constants across a wide range of sites. Carmean (1979b) used linear relationships to convert site index estimates between 13 species of northern hardwoods in the U.S.A. Steele and Cooper (1986) used linear relationships to compare site index estimates for 9 species of conifers in U.S.A. All were positively correlated with slopes varying from 0.3 to 1.8 and intercepts varying from −13 to +14 metres.

Shoulders and Tiarks (1980) examined the influence of rainfall, slope and available soil moisture on the height at age 20 of four species of pines indigenous to that part of the U.S.A. Gulf Coastal Plain. They found that the relative heights of the species were affected by these three characteristics. Where the annual rainfall was less than 1300 millimetres, *P. echinata* was tallest, and where more rain was received, either *P. elliottii* or *P. taeda* was taller, depending on soil, slope and the distribution of the rainfall. *P. taeda* grew best where the cool season rainfall and the warm season rainfall were each less than 650 millimetres. As the warm season rainfall increased and the cool season rainfall decreased, *P. elliottii* exhibited an advantage over the other species.

Equations comparing site indices may be useful for assessing the potential performance of species on sites where they are not present. However, care should be taken with interpretation, as most comparisons published to date compare only the height at index age. Such comparisons should be made only after comparing the methods of determining site index for both species, and comparing the shape of the height growth curves.

All these studies reveal a positive correlation between species specific estimates of site productivity across various sites. A site which is better for species *A* is also better for species *B*, but not necessarily by the same amount. Because of the differing growth habits of various species, comparisons of height or site index based on different species may not give a reliable indication of the timber production potential of various sites. Periodic annual volume increment realized under a specified management regime may provide a better basis for comparison.

3.5.5 Uniformity of Site

A further complexity in assessing the potential productivity of sites arises where the site is not uniform, but is perforated by physical obstructions such as rocky outcrops. An insidious feature of this phenomenon is that conventional measures of site such as site index indicate the potential of the better pockets, and fail to indicate the true average productivity of the site. However, top or predominant height is often determined as the mean of the tallest tree on each of several adjacent non-overlapping plots, and this should reduce bias from such phenomena.

MacLean and Bolsinger (1973b, 1974) proposed the use of stand density index (Curtis 1970), predicted from indicator plants to adjust yield estimates. An alternative approach is to examine the physical attributes of the site and determine an arbitrary reduction, but this approach is not without difficulty (MacLean and Bolsinger 1973b).

3.6 Conclusion

This Chapter has neatly compartmentalized the many options available for site evaluation into discrete categories. This is convenient for the present study, but irrelevant in application. It is likely that the best approaches to site evaluation may employ a combination of several of these options (e.g. Lewis *et al.* 1976).

The development and evolution of an efficient method of site evaluation for TMF will rely on comparisons of alternatives with long term growth recorded on PSPs. Indices such as growth index show promise, but in practice would normally be estimated from stand and environmental variables, including indicator species. Measures of stand height such as maximum stand height, canopy height and the height-diameter relationship may also prove useful, but should be tested against the criteria of Vanclay and Henry (1988):

reproducible and consistent over long periods of time; indicative of the site, and not unduly influenced by stand condition or management history; correlated with the site's productive potential; and at least as good as any other productivity measures available.

4. Construction of Growth Models

Constructing a growth model is not easy, even if good data and reliable site productivity estimates are available. Although some guidelines can be given, a universal step-by-step procedure isn't feasible because too much depends upon characteristics of each specific case. There are many issues involved and this chapter aims to provide a general understanding rather than an intimate knowledge of how to deal with the problems. Help with specific problems should be sought from an expert or from the cited references.

4.1 Model Design

The basic design principles for growth modelling can be summarized: simplicity, logic, modularity and portability. In any project, initial development usually takes less than half the time, the rest is occupied with finding and amending the errors. The problem is not unique to computing, and applies equally to models which are not computer-based. The model should be expressed as simply as possible. Unnecessary complexity introduces errors during model formulation, conceals errors in the computer code, and may not provide better predictions. Many pioneering modelling approaches resorted to a variety of ''tricks'' to reduce computer storage and execution time. However, the relativities of computer costs and staff time have changed greatly, and modellers are urged to strive for simplicity and clarity, even if it is necessary to sacrifice some computational efficiency.

The model should reflect, as far as possible, the actual mechanisms of the growth of the forest. One way to do this is to model the growth in diameter of individual trees, the death of some trees and the recruitment of new trees into the stand. Implementing such a model on a computer involves two aspects: logical algorithms and appropriate data structures (Wirth 1976). Such an approach encourages the creator to think clearly and logically, and often highlights inadequacies in the data and gaps in the knowledge of the forest stands under consideration.

Modellers will frequently be called upon to construct interim components for various parts of their modelling system, because of inadequate data, techniques or knowledge. The model should be constructed in a modular fashion such that each module represents a single process in the total system and is, as far as possible, independent of other components in the system. This approach enables efficient refinement or replacement of the interim components, as more data and new techniques allow improved models to be developed. It should be possible to refine one component equation using new data or analyses without concern for instability in the application of other component equations (Arney 1985).

Growth models may be written in any computer language. Simulation languages offer several advantages as well as disadvantages (Rimmington 1988), but most models continue to be written in Basic, Fortran and C because these are widely available. The language itself doesn't matter, but it is important that the program is clear and logical, so that it is easy to find and fix errors, and easy for others to see what has been done.

Computing technology will continue to develop rapidly. Thus a modeller can be confident that his model will be implemented on several machines during the course of its useful life. Although most versions of computers and computer languages should be ''upwardly compatible'' and industry-standard high-level languages purport to be machine independent, minor changes to the program may be required to implement the model on the new system. Such changes are most frequently necessary to handle data input and report output. This means that the model should be written in standard code without the use of vendor-specific features, and that input-output routines should be confined to a single module separate from the main part of the model.

The application dictates the nature of the output required from a growth model. Initial testing may require detailed output, whereas an analysis of optimal management regimes may involve simulations of several hundred stand-years and produce only brief output at the end. Because of these specific needs, it is advisable to put input-output routines in a separate module and not to embed them in the growth simulation

module. The model should be constructed so that it can be used as a ‘‘black box’’ which grows the stand for one growth period (generally one year but sometimes as much as 10 years e.g. Prognosis). This approach has been used in many growth models and has proved to be a portable and flexible approach.

The following guidelines for growth model design and implementation are adapted from the design criteria of the FORCYTE model (Kimmins *et al.* 1990). A model should be

1. Sufficiently general that it can be applied to a wide variety of stands.
2. Modular with growth modules separate from the management simulator (viz. growth model subroutine separate from control, summary and input/output routines).
3. Able to simulate effects of the major management options.
4. As mechanistic as possible using biologically sound functions rather than empirical surrogates.
5. Driven by operational inventory data rather than by data that require prolonged scientific measurement.
6. Sufficiently diagnostic to permit users to identify errors in data entry, bad data or unacceptable model performance (Each component should provide a basis for rejection).
7. Flexible with options to alter simulation of individual processes.
8. Controlled by the user, with any subjective parameters and ‘‘fudge factors’’ kept to a minimum and amenable to sensitivity testing by the user.
9. User-friendly, with ‘‘plain English’’ reporting that simplifies the interpretation of results.

4.2 Choice of Variables

For best results, the variables used in growth models should not be an arbitrary collection of those correlated with growth or yield, but should be carefully chosen to ensure biologically realistic predictions across the whole range of possible conditions. The meaning, availability, accuracy, correlation and causal relationships of variables should be considered.

4.2.1 Meaning

It is important to understand what a *variable* and a *measurement* really are, and how they relate to the model. Leary (1980) suggested that a measurement, say 25.6 cm on a diameter tape was (Bunge 1967):

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a particular value of
     a numerical variable designating
          a quantitative concept representing
               a property of
                    a concrete system.
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Thus 25.6 represents a value of the variable d.b.h. which designates the concept of girth that represents the property of stem size of a tree (the system). This approach requires that the system is carefully thought through before data acquisition and analysis commences.

4.2.2 Availability

Obviously, if PSP data omit a certain stand variable, then that variable cannot be included in regression analyses leading to the development of a growth model.

Less obvious is the availability of information at the time of application of a growth model. Research plots may often record numerous measurements concerning crown size and shape and stem form. However, if the principal use of a growth model is to project operational inventory data in order to schedule timber

harvests, then only the variables which are (or can be) routinely measured in inventory assessment should be included in the model as explanatory (regressor or independent) variables. So-called independent variables are often not statistically independent, and I shall follow Aitkin *et al.* (1989) in using the terms *explanatory* and *response* variables rather than independent and dependent variables respectively.

4.2.3 Accuracy

One assumption implicit in regression analysis is that the explanatory variables can be measured accurately, i.e. precisely and without bias (Cochran and Cox 1957:16). Any inaccuracy in these variables introduces greater variance in estimates from the equation.

As in any computer application, the results are only as reliable as the inputs. Thus a growth model which employs a variable that cannot be accurately determined is unlikely to provide good estimates of growth and yield. This applies not only to developmental data, but also to data used in applications. For example, internal stem defect may be measured with painstaking accuracy in research data, but since it cannot yet be determined with reasonable accuracy in the field, it should not be used as an explanatory variable in growth models.

4.2.4 Correlation and Cause

A variable should not normally be included in a model unless it is correlated with the response variable. This is generally accepted, and few modellers would include a variable in a model without good reason if it was found to be nonsignificant in standard statistical tests. However, the advent of stepwise regression packages has enabled rather haphazard construction of models. Such packages take a list of potential explanatory variables and find a linear combination which seemingly best describes the response variable. However, such packages do not consider transformations or interactions unless explicitly nominated by the user. A variable found to be nonsignificant in preliminary screening may be significant (or may provide a better fit) if suitably transformed (e.g. using Log(x) rather than x), and the only way to detect such situations is often through careful examination of residuals. A deliberate and carefully reasoned choice by the modeller should result in a better model (i.e. one which provides better extrapolations) than a model empirically selected by a stepwise regression package, although the goodness-of-fit may be the same.

The existence of some correlation between two variables is not necessarily a good reason for inclusion of the explanatory variable in the model. Ideally, modellers should strive to include only causal relationships in their models. In practice, this ideal is rarely attainable (what *causes* tree growth?), but careful choice of explanatory variables is still necessary to ensure biologically realistic growth predictions across the whole range of possible conditions.

An understanding of the biological processes affecting growth and change in trees and forests can help modellers in two ways. It should help to formulate hypotheses and select potentially useful explanatory variables and relationships for the development of models. It is also necessary to properly evaluate and interpret alternative model forms. Any relationship that violates accepted biological principles should be rejected, even if it results in efficient predictions for a particular data set (Hamilton 1990). Such conflicts may be indicative of an error or other anomaly in the database.

Passive monitoring data may indicate greatest growth on the best sites with high standing basal areas, and little growth on poorer sites with little basal area. A growth model constructed from such data might suggest that greater increments accrue in stands with greater competition, as the effects of site productivity and stand density would be confounded. Thus a haphazardly constructed model derived from such passive monitoring data would predict a reduction in diameter increments following thinning, whilst a model from experimental data (e.g. thinning studies) would show an increase in diameter increment. Even if no experimental data are available, the modeller can improve on the haphazard model by deliberate inclusion or exclusion of terms in the model (e.g. by excluding the contradictory site productivity term, even if significant, to ensure sensible extrapolation).

4.3 Strategies for Grouping Data

In developing models for mixed forests, there may not be sufficient data for a particular species to develop a reliable relationship. In this situation two approaches may be adopted. One can guess at the appropriate relationship, and employ this subjective relationship in the model, or data can be aggregated so that a relationship can be developed for a group of species. Subjective relationships are often used in succession models where it is important to maintain individual species identities, but not necessary to accurately predict yields. Subjective relationships pose problems of bias, and it is preferable to use an objective method for yield prediction models. Aggregation also enables the number of equations required for modelling to be reduced to a manageable number. Such an approach should aggregate species that are in some sense similar so as to minimize within-group variability and maximize the variation between groups.

There may be some temptation to use economic criteria to identify groups. Although this may be useful in assessing the present value of a forest stand, it is a potentially dangerous approach in modelling as it may bear little relevance to the dynamics of the species.

In TMF, there are often a few species which are difficult to discriminate and are often confused. Combining these similar species may reduce overall errors by eliminating across-group species confusions (confusion of species within groups would not influence predictions). Not only will this aid the rapid collection of data, but it also eliminates a serious source of error in the data. If the species comprising any group are in fact, frequently confused, then the variation in the grouped data will not be appreciably greater than in the individual species data, even if the growth patterns vary considerably. It may be possible to classify trees according to their regeneration strategy and growth rate. The light demanding pioneer species require large gaps in the canopy to become established, and tend to have rapid growth rates and small long-lived seeds. The shade tolerant species require only small gaps in the canopy to regenerate, and are often slow growing with large short-lived seeds. Alder (O.F.I., *pers. comm.*) suggested that in TMF, fast growing species account for most of the observed increments greater than 8 millimetres per annum, whilst the slower species generally grow less than 4 millimetres per year. Such a natural division could form a good basis for grouping species for modelling. However, the data for TMF in Queensland do not support such a division into fast and slow growing species.

Swaine and Whitmore (1988) proposed grouping into pioneer and climax guilds based on seed germination and seedling establishment strategies. They listed 16 growth characteristics which are unique to, and often present in pioneer species. These include copious well distributed seed and rapid height growth. Swaine and Whitmore (1988) recommended that further classification should be based on height at maturity (pygmy, small, medium and large). However, these characteristics may not be known for all species, and may not provide a good indication of diameter increment pattern.

Shifley (1987) aggregated species subjectively on taxonomy and silvicultural characteristics. Where little is known about the characteristics of a species, the botanical affinity of a species may give a useful indication of the possible behaviour of a species, and the logical grouping to adopt. However, taxonomy need not provide a reliable basis for grouping; for example the genus *Eucalyptus* includes both the world's tallest hardwood trees (*E. regnans*) and mallees (e.g. *E. vernicosa*) which may barely attain 2 metres height at maturity. Vanclay (1991b) also demonstrated significantly different diameter increment patterns for several species of the same genus in Queensland TMF.

The many species occurring in the TMF pose a number of difficulties for subjective grouping: how many groups are necessary, and how little-known species should be assigned. Meldahl *et al.* (1985), Leech *et al.* (1991) and Vanclay (1991b) examined objective procedures to resolve these questions. Meldahl *et al.* (1985) argued that the grouping should reflect the dynamics of growth, and that this could be best expressed through the coefficients of a regression equation on diameter increment. They attempted cluster analysis (Gower 1967) on these coefficients, but found that reasonable results could be obtained only when the regression analysis was constrained to a single explanatory variable. They investigated several possible explanatory variables, and obtained best results when diameter increment of each individual tree (DI) was predicted from the basal area of trees larger than the subject tree (BAL):

$$\hat{DI} = \beta_0 + \beta_1 BAL$$

Meldahl *et al.* (1985) used cluster analysis weighted by the inverse of the significance level of β_1, and obtained twenty clusters from 110 species-type equations. The number of data assigned to each cluster

varied greatly, and the outcome was subjectively adjusted to provide the final grouping. The adequacy of final groups was tested by fitting a multiparameter linear function and examining the total (across clusters) residual sums of squares, on the assumption that a better grouping would result in a better fit. Whilst the method provided a satisfactory grouping of similar elements, it did not provide a unique solution.

Leech *et al.* (1991) used a Behrens-Fisher analogue of Hotelling's T^2 to group 27 species for volume equation estimation. They used a polynomial equation to predict tree volume (Y) from tree diameter (D) for tree i:

$$Y_i = \beta_{0i} + \beta_{1i}D + \beta_{2i}D^2 + \cdots \beta_{ni}D^n$$

Then, representing the vector of coefficients as

$$u'_i = [\beta_{0i}, \beta_{1i}, \beta_{2i}, \cdots \beta_{ni}],$$

Hotelling's T^2 between two species i and j can be defined as

$$d_{ij}{}^2 = (u_i - u_j)'S^{-1}(u_i - u_j)$$

where S^{-1} is the combined covariance matrix of regression coefficients for species i and j. The T^2 statistic is similar to the uni-variate linear discriminate function. By calculating all possible combinations a symmetric matrix with zero diagonal elements can be formed. Principal coordinate analysis (Gower 1966) was used to group species on the basis of this matrix. Leech *et al.* (1991) concluded that the technique should only be used when the order of the polynomial, and the sign of the highest term, were the same for each of the two individual species equations. The method was also computationally intensive.

Vanclay (1991b) used pairwise comparisons between species specific diameter increment regressions. His method involved the following steps:

— Ranking species in order of increasing number of observations.

— Assigning the species of highest rank the founding species of group 1.

— For each species in decreasing order of rank, conducting pairwise F-tests with the founding species of higher rank. If the incoming species was significantly different ($P<0.01$) from all existing founding species, it became the founding species of a new group. Species not significantly different from all founding species remained ungrouped.

— After identifying all founding species, those species remaining ungrouped were compared, in order of rank, with all existing groups, and grouped with the most similar group. Similarity was determined as that grouping which lead to the smallest increase in residual sum squares when the incoming species was amalgamated with the group. These comparisons were made with the whole group, not just the founding species.

This approach overcomes many of the difficulties associated with the alternatives discussed above, and is computationally efficient. Instead of a comparison of all possible pairs, initial comparisons are made between species with many data, reliable parameter estimates and homogeneous variance. Species with few data are only later compared with one of these major groups. It also avoids the need to arbitrarily select a subset of the more numerous species to define the groups. This selection is by no means intuitive as in Vanclay's (1991b) study the species ranked 186 with only 13 observations initiated a new group. There is, unfortunately, no guarantee that the outcome is optimal, and the grouping thus derived is specific to the particular data set and increment function used. Despite these weaknesses, it provided a useful classification of 237 species into 41 groups for the development of a growth model for TMF in Queensland.

The appropriate approach to use depends on the quantity and quality of data. The methods reviewed here all have weaknesses which have yet to be overcome. Where few data are available, a subjective classification based on growth habit may be best. If sufficient data are available, either Vanclay's (1991b) or Leech *et al.*'s (1991) method should be attempted, noting Meldahl *et al.*'s (1985) conclusion that simple models provide more robust results.

Vanclay (1991c) demonstrated that an aggregation based on diameter increment may not be suited to modelling mortality, and that whilst it may be necessary to aggregate species to formulate group equations, it may be best to retain individual species identities during growth and yield simulations (e.g. Vanclay and Preston 1989).

4.4 Choice of Equation

4.4.1 Empirical Equations

Empirical equations are expressions which describe the behaviour of the response variable without associated hypotheses of causation or explanation of the phenomenon. This does not mean that empirical functions do not provide biologically realistic predictions, nor does it mean that they are inferior to supposedly theoretical equations. They can and should be formulated to behave in a biologically realistic way across a wide range of possible conditions. The most widely used form of empirical equations, and the only form considered here, is the linear equation. Linear equations are those in which the explanatory variables enter in a linear fashion:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots$$

where Y is the response variable (e.g. diameter increment), X_1, $X_2, \cdots$ are explanatory variables (e.g. tree diameter, site productivity, stand basal area), and β_0, $\beta_1, \cdots$ are parameters to be estimated. These equations are probably the most widely used in growth and yield studies. There are several advantages in using linear equations contributing to their wide use. Most computers and many pocket calculators have reliable packages which enable such equations to be fitted to data. The solution to the equations is unique, easily obtained and the confidence interval about the equation is easily determined.

However, there are also disadvantages with such empirical equations. They often do little to further understanding of the processes involved in the system being modelled. While they may describe the data satisfactorily, they may (unless carefully formulated) give totally anomalous estimates for values lying outside the range of data on which they are based. Careful choice of the variables used and transformations applied will do much to overcome these problems.

4.4.2 Theoretical Equations

In contrast to empirical equations, theoretical equations have an underlying hypothesis associated with the causation or explanation of the phenomenon described by the response variable. There are few theoretical equations formulated expressly for forestry applications. Most theoretical equations have been borrowed from other disciplines, and as a result are more empirical than theoretical for some forestry applications. However, some general principles govern the behaviour of many systems, and provide the basis for these theoretical equations.

Systems theory is an expression which was coined by Bertalanffy (1945, 1968), and the approach has been used increasingly as an effective technique for problem solving (Bakuzis 1971). Systems theory simply means ‘‘identifying correspondences in principles that govern the behaviour of entities that are intrinsically widely different’’ (Bertalanffy 1968:32). Bertalanffy (1941, 1942, 1957, 1968:182) hypothesized that the growth of an organism could be represented as the difference between the synthesis and degradation of its building materials. He assumed that the processes of anabolism (synthesis) and catabolism (degradation) could be expressed as allometric functions of weight (W), and thus growth (dW/dt) would thus approximate

$$\frac{dW}{dt} = nW^m - pW^q$$

where m and q are the constants of anabolism and catabolism respectively. He argued that catabolism was generally directly proportional to weight in animals, and that in any case, small deviations in q from unity would have little effect on the form of the relationship. Thus he proposed

$$\frac{dW}{dt} = nW^m - pW$$

Bertalanffy (1957, 1968:184) argued that m could vary between 2/3 to 1, but would generally be 2/3 for the higher animals. He did not propose any specific value of m for plant individuals or populations. Turnbull (1963) appears to have been the first to use the Bertalanffy equation to model forest stands. The application

of the equation and development of its integral form are discussed by Turnbull and Pienaar (1966) and Pienaar and Turnbull (1973). The Bertalanffy equation has been widely used in forestry (e.g. Moser and Hall 1969, Murphy and Farrar 1982, Shifley and Brand 1984). It was used by Richards (1959) for plant growth studies, and by Chapman (1961) in studying fish populations, and is often known as the Chapman-Richards equation in north America (e.g. Pienaar and Turnbull 1973, Shifley and Brand 1984, Zeide 1989, 1990).

The Bertalanffy equation is a member of a family of asymptotic, nonlinear growth equations (Rawat and Franz 1974) including:

Monomolecular growth (or Mitscherlich) equation:

$$Y = A\left(1 - e^{-kt}\right) \qquad \frac{dY}{dt} = k(A - Y)$$

Autocatalytic growth (or Logistic) equation:

$$Y = \frac{A}{1 + be^{-kt}} \qquad \frac{dY}{dt} = \frac{kY(A - Y)}{A} = kY - aY^2$$

Gompertz growth equation:

$$Y = A \times e^{-be^{-kt}} \qquad \frac{dY}{dt} = kY \times Log\left(\frac{A}{Y}\right)$$

Bertalanffy equation:

$$Y = A\left(1 - be^{-kt}\right)^c \qquad \frac{dY}{dt} = nY^m - pY$$

where Y is the size of the organism, A is the asymptotic maximum size, t is time, and b, c and k are constants. Zeide (1990) explored the relationships between these and other similar equations, and expressed them in alternative (difference and division) forms (e.g. Monomolecular: $kA - kY$ or kA/e^{kt}).

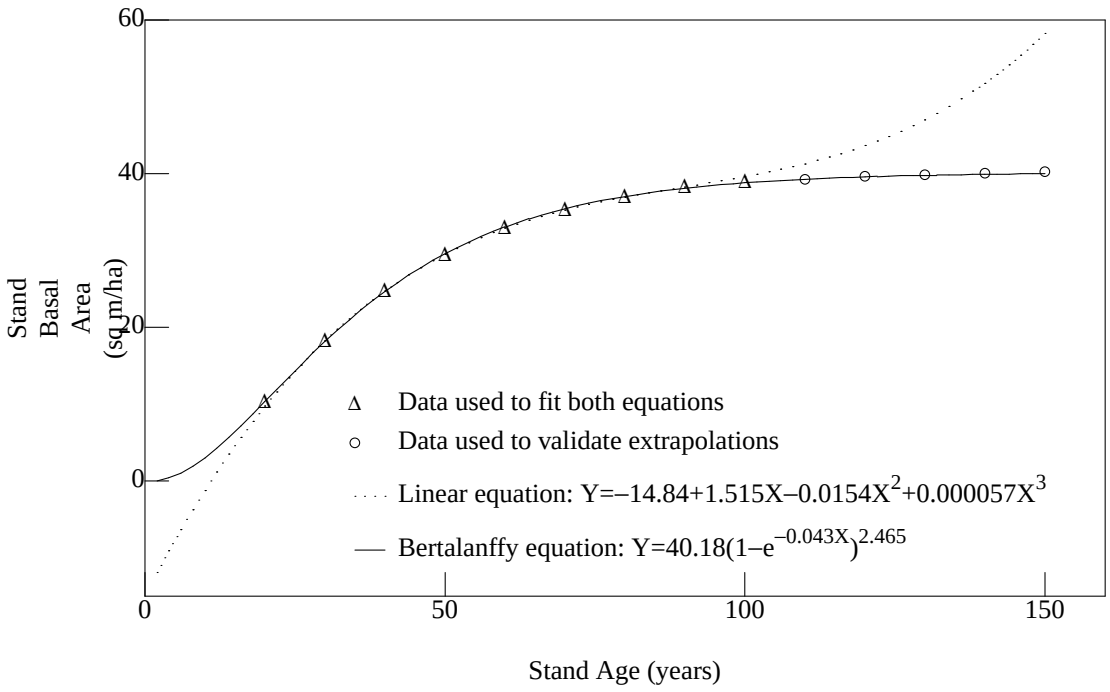

Figure 7. Dangers of Extrapolating Empirical Models (Payandeh 1983)

Bailey (1980) suggested a new equation which has no biological basis, but is an extremely flexible function, describing a family of asymptotic curves which encompass both the Bertalanffy and the Weibull (1951) equations:

$$Y = A\left(1 - e^{-bt^d}\right)^c$$

Martin and Ek (1984) found that carefully formulated empirical equations could be more accurate than theoretical equations for a wide range of data. However, they considered that theoretically based equations may be more reliable for predictions extrapolating beyond the range of the data. Payandeh (1983) gave some examples illustrating the dangers of extrapolating empirical models. Figure 7 illustrates one such comparison between an empirical polynomial equation and the Bertalanffy equation. However, the polynomial equation in this example is not deliberately formulated, and better empirical equations are possible (e.g. an asymptotic equation $Y = \beta_0 - \beta_1/X$).

Equations can be empirical or otherwise, but should be carefully formulated to provide sensible predictions. Kowalski and Guire (1974) emphasized that ‘‘finding a function that makes biological sense has much more to recommend it than searching for a function that will provide only a close mathematical fit. Mere goodness of fit is no justification for adopting a given model since several functions may fit the data equally well.’’

4.5 Regression Techniques

There are a number of techniques available for fitting equations to data, and the appropriate technique depends largely upon the nature of the data and the chosen equation. There is one rule which holds for all these approaches: plot the data and the fitted model. At least three series of graphs should be prepared and examined: one with the raw data showing the response variable plotted against several of the explanatory variables, one showing the raw data and the fitted model overlaid on the same graph, and one showing the fitted model evaluated for a wide range of explanatory variables. If possible, the fitted model should also be compared with independent data. Such graphs remain one of the best ways to detect errors in and inconsistencies between the raw data and the fitted model.

4.5.1 Hand-drawn Curves

Hand drawn curves may be useful, especially where there are few data. However, care should be taken when compiling hand-drawn curves to minimize bias. Mosteller *et al.* (1981) found that hand fitted lines tend to follow the first principal component rather than the least squares fit (i.e. minimize the perpendicular distance from the data points to the fitted line, rather than the vertical distance as in least squares). The effect of this generally will be to overestimate the gradient, thus (assuming a linear response such as a volume-basal area line) underestimating the response for small trees and overestimating for large trees. If regression analysis is subsequently used to estimate parameters of an equation describing the hand-drawn line, remember that the goodness-of-fit statistics reflect the points chosen from the hand-drawn curve, not the goodness-of-fit to the original data, and will thus provide an over-optimistic estimate of the quality of the fit, and underestimate its variance.

4.5.2 Ordinary Least Squares Linear Regression

Linear regression refers to the situation discussed above in which explanatory variables enter the objective function in a linear fashion. It in no way implies that the resulting relationships are straight lines. This form of regression is probably the most widely used method of fitting equations to data, and some form of linear regression package is available on most computers and many calculators. Three approaches can be distinguished.

The first is the normal situation where the user specifies the response and all the explanatory variables, and the computer merely determines the values of the required coefficients. This approach encourages clear and logical thinking, as the user must decide which explanatory variables will best describe the response variable. Plotting the raw data and the residuals from the preliminary models against potential explanatory variables will assist the formulation of the final model.

Stepwise methods of regression analysis leave the choice of variables to the machine. The user identifies the response variable and provides a list of possible explanatory variables. Step-up regression, or regression by stepwise inclusion (or forward selection) starts assuming that the response variable is adequately described by the mean, then iteratively includes additional explanatory variables until the remaining variables fail to meet the required level of significance. Step-down regression, or regression by backward elimination, adopts the opposite approach and starts with all variables included in the model, and iteratively deletes variables which fail to reach the required level of significance. The latter method fails, and the former method may be unreliable, if there is much multicollinearity (correlation between explanatory variables) in the data. Step-up and step-down regression do not necessarily lead to selection of the same subset of possible explanatory variables. Multicollinearity is often a problem in growth modelling data, and my own experience suggests that it is useful when using step-up regression to repeat the analysis with the variable which was selected first in the initial analysis removed from the selection set on the second occasion. Although step-up regression does not necessarily lead to the ''best'' subset, it may work well where multicollinearity is modest (Snedecor and Cochran 1980:360). Regression by leaps and bounds (Furnival and Wilson 1974) may overcome the disadvantages of both stepwise approaches, but is rarely available in statistical analysis packages.

Combinatorial screening (Grosenbaugh 1967) is an approach which fits every possible combination of some or all the explanatory variables. This approach is not without shortcomings. It will find the best combination of variables specified, but the possibility always exists that a better model could be constructed using different transformations of, or interactions between, the explanatory variables. In addition it is computationally intensive, and overlooks the role of the deliberate selection of variables and transformations in ensuring sensible predictions. However, it does overcome some of the limitations of stepwise procedures (Grosenbaugh 1958, 1967).

Ultimately, however, the method of obtaining a model is irrelevant. The important thing is whether or not the model will provide useful predictions, assessed by an appropriate suite of diagnostic tests. Prominent among these criteria is the requirement that the model provides biologically reasonable predictions for the whole range of possible conditions.

4.5.3 Nonlinear Regression

Nonlinear regression enables the fitting of models in which the estimated parameters do not enter in a linear additive manner. Many theoretical (e.g. Bertalanffy) and asymptotic models (e.g. $Y = a - b/(c + X)$) are of this form. Whilst nonlinear regression allows great flexibility in formulating models to ensure sensible extrapolation, it does have some limitations. One problem is that, unlike linear regression, nonlinear regression does not necessarily provide a unique best unbiased solution for a given set of variables. Nonlinear regression is iterative, and the solution may be influenced by the starting conditions specified by the user or estimated by the package. One further problem is that there is no general theory for confidence interval estimation (Conway *et al.* 1970).

Nonlinear regression can be solved by any of several methods (e.g. steepest descent, Gauss-Newton, Marquardt's compromise) available in standard computer packages (e.g. BMDP, SAS, SPSS). Supplying analytical derivatives rather than using computational approximations usually results in more efficient and more precise parameter estimation. If the first attempt at fitting a non-linear equation fails, you should re-examine the data carefully for errors and outliers, try alternative initial values, and then try some other method or statistical package. The user should always perform some sort of sensitivity analysis to ensure that the global minimum has been reached, as some situations may lead to a local minimum rather than to the ''best'' fit.

Ratkowsky (1983, 1990) discussed why problems arise in iterative estimation of the parameters of nonlinear models. These can usually be overcome by ''reparameterizing'' the model so that the model behaves in a ''near to linear'' fashion. The simplest method of iterative estimation, the Gauss-Newton method, can then be employed and the resulting parameter estimates are unbiased, normally distributed, minimum variance estimators. If the model does not behave in a near to linear fashion, the parameter estimates will not have these desirable properties and the complex iterative estimation techniques mentioned above may be necessary. Reparameterization involves expressing the parameters of one model as a function only of the parameters of another model. For example, the models

$$Y = \frac{p_1 X}{X + p_2}$$

and

$$Y = \frac{X}{q_1 X + q_2}$$

where p_1, p_2, q_1 and q_2 are parameters, are reparameterizations of each other since it can be shown easily that $p_1 = 1/q_1$ and $p_2 = q_2/q_1$. The model is not changed in any way by this reparameterization and its fit to the data will be exactly the same in either form. However, the statistical properties of the parameter estimates may be very different depending on the form that is used.

Ratkowsky (1983, 1990) discussed the methods that can be employed to find suitable parameterizations of particular nonlinear models so that they behave in a near to linear fashion. The more recent handbook (Ratkowsky 1990) described a wide range of nonlinear models which have been used commonly in science, their shapes and their statistical estimation properties.

4.5.4 Techniques for Multiple Measurements

Ordinary least squares (OLS) regression techniques assume that all observations on any variable are independent of each other. In forestry data sets, this is often not the case and it is common to find that two or more measurements have been taken from each of a number of sampling units. The ‘‘sampling units’’ in this context might be individual trees on each of which a number of measurements have been made (possibly at different positions on each tree), or they might be individual plots in each of which many trees have been measured. The most common instance of this in growth modelling studies involves individual sampling units (e.g. trees or plots) which have been remeasured several times (i.e. in different years). These repeated measurements on the same trees tend to be correlated, and are thus not statistically independent.

When OLS regression is applied to such data sets, the estimator of parameters is unbiased, but the estimators of the covariance matrix of the parameter estimates and the residual variance of the regression equation are biased towards underestimates. It is inappropriate to use ordinary least squares regression to estimate the parameters of a growth model with data containing repeated measurements from individual sample plots, since this method will tend to underestimate the variance of the parameter estimates (Davis and West 1981). This means that it is not possible to carry out properly the normal statistical hypothesis tests that are usually necessary to apply the results of regression analysis.

West *et al.* (1984) reviewed how and when this problem arises and a number of solutions to the problem which have been suggested. None of these solutions is entirely satisfactory. Subsequently, West *et al.* (1986) developed a new solution which appeared to have practical application, but was rather too complex for routine use. Moser *et al.* (1990) discussed alternatives for analyses of variance studies. None of these alternatives is entirely satisfactory and only experience can suggest the most appropriate solution for a particular data set. In some cases such as when the number of multiple measurements is small in comparison with the number of sampling units, the problem may not be serious. The simplest solution may be to use only one observation from each sampling unit. OLS regression is then appropriate and the problem is avoided altogether, but information is lost by using only part of the full data set. An alternative may be to use only one observation per sampling unit to formulate the model and test for the significance of terms, and having resolved the model, to recalibrate it using the full data set.

In an empirical study of yield models, Borders *et al.* (1988) found no serial correlation when data were derived from non-overlapping growth intervals, and that problems associated with serial correlation may be model dependent. The specific problem of serially correlated errors appears worst when yield (i.e. total standing volume) is predicted from age. When growth is predicted from initial size, as is common in models for TMF, these problems may be reduced and OLS may be acceptable. If OLS is used despite serially correlated errors, it will provide unbiased parameter estimates, although the variance will be underestimated. The implication of this is that OLS may give rise to parameter estimates which appear significant, but which in reality are not, leading to a more complex model than is warranted. If in doubt, use a higher level of probability for statistical tests (e.g. use P=0.01 rather than P=0.05); unfortunately no simple guide to the adjustment required can be given.

4.5.5 Robust Regression Techniques

Robust regression refers to a number of techniques which attempt to overcome the problems of a non-normal error distribution and correlated explanatory variables. Unusual data points often occur in growth data, and although they may have an excessive effect on the least squares estimates of coefficients, they cannot validly be omitted from the analysis. None-the-less, screening of data and the removal of outliers is fairly standard practice amongst biometricians (e.g. Arney 1985). This ‘‘massaging’’ of data not only influences the estimates of coefficients and variances, but also may mean that the analysis cannot be reproduced by another worker.

Schreuder *et al.* (1979) identified three classes of robust estimators, and observed that of these, maximum likelihood estimation was most amenable to regression analysis. They demonstrated the effect of using three approaches of maximum likelihood estimation, with both relatively clean and noisy data. All three techniques gave similar estimates to OLS with the clean data, but the estimated parameters varied considerably when the techniques were applied to the noisy data.

Robust estimators are a compromise between including all the noisy data, and using only the massaged data. One advantage of massaging data is that it forces the researcher to critically evaluate the data and decide if it comes from the population of interest. Choosing the parameters which determine the degree of robustness in maximum likelihood techniques does not force the same decisions upon the researcher. Since the outcome from robust techniques depends largely upon the subjective determination of scale parameters, these techniques remain an investigative tool for the data analyst and are not a cure-all for bad data (Hamilton 1979).

For most growth modelling applications a robust and reliable result can be obtained by fitting a carefully formulated linear equation using ordinary least squares linear regression. Whilst this approach makes some assumptions unsatisfactory to theoreticians, it is relatively robust, repeatable and readily available.

4.6 Statistical Assumptions

The basic assumption of least squares analysis is that the errors are independently and identically normally distributed. This assumption should be satisfied for hypothesis testing and calculation of confidence intervals. The effects of violating this and other assumptions may or may not be serious, depending upon the situation, but the dangers are greater when the user is unaware of the possible consequences.

The best way to check data, fitted models and statistical assumptions is to plot the response variable against several explanatory variables, with and without the fitted model, and to plot the residuals against explanatory variables and the fitted values. Wilson (1979) reviewed techniques for examining and interpreting these residual distributions.

4.6.1 Independent Observations

In statistical analyses, the observations on the response variable are assumed to be independent. This assumption may be violated in time series data, such as when increments are obtained from several remeasurements of permanent plots. The variability of the information obtained is reduced because several observations have been made of the same individual. This has little effect on the estimation of the regression coefficients, but results in underestimates of their variance, which leads to the acceptance of complex models containing more parameters than is warranted. Methods for analysing such data have been discussed in Section 4.5.4.

4.6.2 Independence of Explanatory Variables

Collinearity between explanatory variables frequently exists in forestry. An example of this is volume estimation from height and diameter, where height and diameter are clearly correlated. However, the

problem is not always so obvious. Where all the explanatory variables appear to have little correlation, a linear combination of some variables may be highly correlated with one of the remaining variables, and the problem still exists. Another instance may arise in lists of transformed variables for stepwise regression, where for example, $Log(DBH)$ is perfectly correlated with $Log(BA)$ (i.e. $Log(BA) = Log(kDBH^2) = Log(k) + 2Log(DBH)$). Apart from the problems of stepwise regression already discussed, such correlation also leads to numerical problems in parameter estimation. The absolute value of the estimated parameters may not only be too large, but the sign may be wrong, and the parameter may change substantially after the addition or deletion of a single data point. This situation exists where the computations are exact, and the result of even small rounding errors in the computer may make parameter estimates even more unreliable. Correlated explanatory variables may lead to substantial overestimates of the F-ratio, and t-tests, and thus to the acceptance of models containing more parameters than warranted.

Large variance inflation factors (Marquardt 1970) may indicate correlated explanatory variables which may contribute to poor extrapolations. These problems can be partially overcome by the use of ridge regression (e.g. Marquardt 1970, Snee 1973, Hocking 1976), but it too is a compromise and introduces other problems (Warren 1979). Fortunately, multicollinearity does not inhibit our ability to obtain a good fit, nor does it affect inferences about mean responses or predictions from new observations, provided that these inferences are made within the region of observations (Neter and Wasserman 1974:341).

4.6.3 Homogeneity of Variance

In heteroscedastic data the variance is not constant across the range of explanatory variables. This occurs frequently in forestry, and the classic example is the volume line in which trees of small basal area have very similar volumes, but the volumes of the trees of large basal areas may vary considerably. Because of the influence of these extreme residual values on the least squares estimates, they may disproportionally influence the form of the equation. Realistic confidence regions about the equation cannot be computed by the usual formulae. Tests for the homogeneity of variance are discussed by Snedecor and Cochran (1980:252).

Two solutions to the problem exist. A variance stabilizing transformation can be used, or a variance estimating function can be used to weight the regression. Weighted regression assumes that the errors are distributed normally with variance ($v \times \sigma^2$), and that a function to predict v can be determined. The weights used in the regression are the inverse of this function ($w = 1/v$).

Several transformations may be used to stabilize the variance, and the appropriate one to use depends on the nature of the data. Suitable transformations can be determined analytically (e.g. Box and Cox 1964, Snedecor and Cochran 1980:288) or graphically. If the variance is plotted against the mean, a horizontal line ($\sigma^2 = c$) indicates that no transformation is required; a straight line increasing with the mean ($\sigma^2 = \mu$) suggests a square root transformation and a line curving upwards ($\sigma^2 = \mu^2$) a logarithmic transform. Alternatively, the slope of the line fitted to $Log(\sigma^2)$ on $Log(\mu)$ also indicates the appropriate transformation. If the line passes through the origin, a slope of 1 indicates that a square root, slope 2 a logarithmic, and slope 4 that a reciprocal transformation should be used. If such a transformation has been used, predictions will contain transformation bias, the magnitude of which depends upon the variability of the data. Often this bias may be small enough to be ignored. However, where a poor fit is obtained, the user should consult a standard text for details regarding the removal of this transformation bias when performing the back transformation (e.g. Kendall and Stuart 1968:95, Bradu and Mundlak 1970, Snedecor and Cochran 1980:287). Weighted regression avoids the need for such transformations and corrections.

Transformations not only affect the distribution of errors, but also influence the explanatory variables. For example, a linear model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \cdots$ assumes an additive interaction between the explanatory variables (X_i), whereas the transformed model $Log(Y) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \cdots$ implies a multiplicative effect which may or may not be appropriate. Weighted regression also overcomes this difficulty.

4.6.4 Normally distributed Errors

The effect of non-normal data depends largely upon the nature of the error distribution. Many standard

statistical tests are sensitive to long-tailed distributions, and asymmetrical distributions may lead to biased estimates. Non-normal data may lead to inappropriate confidence regions about the regression surface if the usual formulae are used.

Transformations may be used to correct departures from normality in data. Counts of rare events such as numbers of defects tend to be distributed in a Poisson fashion, and can often be stabilized by a square root transformation (Snedecor and Cochran 1980:288). Data concerning proportions are often binomial, and can be stabilized by the angular (arc sine square root) transformation (Snedecor and Cochran 1980:290). The truncated distribution of bounded count data may be improved by the logit transformation: $Y' = Log(Y/(A - Y))$, where Y lies between zero and A, the upper bound.

4.6.5 Free of Outliers

Outliers in the data are observations which deviate greatly from the general trend. They may have a great influence on least squares estimates and, if due to errors, may lead to biased predictions. Since not all outliers are caused by error, they should not be deleted from the data set without careful investigation of their source and validity. The robust regression techniques discussed above may relieve the problems of outliers, but also introduce new problems of interpretation. All suspect data should be critically evaluated to decide whether they are valid and from the population of interest.

Diameter increment data need special care when examining outliers, as many decrements caused by measurement error will be associated with an excessively large increment in the preceding or subsequent period. These negative outliers should not be removed from the analysis without removing the corresponding large increment, or bias will result. Not all decrements are caused by measurement error. Some may be caused by natural variation in stem size or by bark shedding in some species, and should not be removed from the analysis. These factors re-emphasize the importance of using a remeasurement interval sufficiently long so that the diameter increment is large relative to the measurement error and to fluctuations in stem size.

Not all erroneous data reveals itself as large outliers. Extreme values may exert great influence over regression results without producing a large residual. Influence values and leverage plots (Aitkin *et al.* 1989:102) enable such values to be detected. Observations with large residuals or influence values should be carefully examined and checked against the original field sheets. They should not be automatically rejected, but should always be carefully evaluated.

4.6.6 Accuracy of Explanatory Variables

Standard least squares regression assumes that the explanatory variables are accurate and have no error and no bias. However, in forestry applications, the explanatory variables frequently contain some measurement error, and they may also be rounded. The effect of this is to increase the variance of the estimates beyond the values predicted by the usual formulae. Thus the calculated confidence region will be conservative. Provided that the range of each explanatory variable is large relative to its error, these errors can effectively be ignored and the usual least squares analysis may be performed (Draper and Smith 1981:122).

4.6.7 Precision of Response Variable

It is assumed that the response (or dependent) variable is continuous, when in practice it may be measured with finite precision and may take only a finite number of discrete values. If the precision of measurement is low relative to the standard deviation of the fitted model (i.e. $\delta/\sigma > 0.1$) (Aitkin *et al.* 1989:322) as is often the case with diameter increment data, then this discreteness should be allowed for explicitly in the model. A model incorporating the discrete representation of the probability distribution can be fitted using maximum likelihood, but is considerably more complicated than the usual procedures. Failure to account for the discrete nature of the response variable will lead to inflated estimates of variance and the adoption of simpler models than would otherwise be warranted. To increase precision of diameter increment measurements, trees on PSPs should be measured to the nearest millimetre, and the remeasurement

intervals should be sufficiently long to allow at least one centimetre of increment to accrue.

4.7 Tests for Comparing Equations

Some difficulty arises in deciding what test statistic to use in comparing alternative functions developed with the same set of data, or comparing functions developed by different researchers. The most widely-used criterion, the coefficient of determination (r^2) and similar statistics including the correlation coefficient (r) and the ratio of residual mean squares (r'^2), have several limitations. The coefficient of determination may be derived from $r^2 = 1 - RSS_m/RSS_1$ where RSS_m is the residual sum squares of the model under consideration, and RSS_1 is the residual sum squares about the mean. This expression illustrates two advantages of r^2: it is a ratio, and thus independent of units, and it is an indicator of how the model compares with a simple average ($r^2 = 0$) and with the perfect fit ($r^2 = 1$). However, it is not necessarily possible for $r^2 = 1$ to be attained, as the data may contain ''pure error'', or natural variation, which cannot be accounted for by any model. Warren (1981) gave an example where r^2 was close to unity, but the model was obviously a poor fit to the data.

Other disadvantages of r^2 are not so obvious, but can be easily demonstrated. It takes no account of the number of terms in the model, and more terms, even if nonsignificant, result in a higher r^2. Snedecor and Cochran (1980:359) recommend the use of Mallows' (1973) criterion (C_p) which is a measure of the expected variance associated with predictions from the fitted model. Because Mallows' criterion accounts for the error in estimating each parameter, it leads to the acceptance of simpler models than many other decision criteria. However, there may be some difficulty in estimating the residual population variance of different populations with equal accuracy.

A further problem with r^2 can best be demonstrated by example. Consider a regression of two highly correlated variables, say stem volume over and under bark. The model $VUB = VOB \times f(D)$ will usually yield a very high value of r^2. Because of the heteroscedastic nature of these types of data, the model $VUB/VOB = f(D)$ is more appropriate, but yields a much lower r^2. In both cases the same function $f(D)$ may have been used, the difference being the assumed error distribution ($N(0, \sigma^2)$ and $N(0, \sigma^2/VOB)$ respectively). For these reasons, yield equations ($D + ndD/dt = f(D)$) and basal area increment equations ($dkD^2/dt = 2kDdD/dt = g(D)$) often indicate higher r^2 than diameter increment models ($dD/dt = h(D)$) fitted to the same data.

The Furnival (1961) index overcomes this problem by expressing the average standard error in the original untransformed units, and is derived from the maximum likelihood estimate. It can be computed as $I = [f'(Y)]^{-1}S$ where $f'(Y)$ is the derivative of the transformed response variable with respect to the untransformed variable, and S is the usual estimate of standard error. Furnival (1961) gave an example where Spurr's (1952) combined variable volume equation $V = a + bD^2H$ yields an r^2 of 0.96 and a Furnival index of 19.2, and an alternative formulation $V/(D^2H) = b + a/D^2H)$ yields $r^2 = 0.72$ and Furnival index 9.4. The alternative formulation is the preferred model, as indicated by the distribution of residuals and by the Furnival index (smaller is better), but this is not suggested by r^2.

Whilst r^2 may give a reliable indication of the fit of the model to the development data, it gives an over-optimistic indication of the model's predictive ability. The Prediction Sum of Squares (PRESS) (Allen 1971, Aitkin *et al.* 1989:134) indicates the predictive ability of the equation, estimated by cross validation. This entails omitting each observation in turn from the data, fitting the model to the remaining observations, predicting the response for the omitted observation, and comparing the prediction with the observed value. PRESS is provided by many statistical packages. Since PRESS is analogous to RSS, small values are desirable, and the cross-validation r^2_{CV} can be computed.

Warren (1986) discussed the role of these and other tests in the interpretation of analyses, emphasized the role of reason rather than ritual, and gave suggestions regarding the choice of significance level. In comparing alternatives, consider the Furnival index and PRESS as well as r^2. Don't be pre-occupied with these indices, but consider if predictions will be biologically reasonable over a wide range of possible values for the explanatory variables, and if the coefficients are reasonable estimates of the effects of the individual terms (Snee 1977).

4.8 Special Considerations in Stochastic Models

The concept of mean and variance are well established in experiment methodology, and few researchers would formulate policy decisions based solely on the mean experimental result. Most researchers and managers consider not only the mean but also the variance of estimates. Similarly, decision making based on risk appraisal as well as other economic criteria is a well established principle.

However, in growth modelling for timber yield prediction, this appraisal of variability and risk is generally lacking. Most modelling methodologies developed for timber yield prediction are deterministic. The few models which are stochastic have been developed because of the inability of the modeller to satisfactorily resolve some function, and not to evaluate the variation in estimates of timber yield. This seems naive on the part of modelers and resource managers, particularly when it is not uncommon for forest owners to make long term commitments for timber supply, and where a failure to meet this commitment may be embarrassing to the owner.

It is unlikely that deterministic models will be replaced by stochastic models; deterministic models have proven their utility and cannot be replaced by stochastic models. Deterministic models are more efficient at predicting the mean response, and can be used to efficiently determine the optimum management strategies for forest stands in a way not possible with stochastic models. Compatible deterministic and stochastic models may prove useful for forest management. Alternatively, variance approximation may be used to deterministically estimate the variance of predictions (Mowrer and Frayer 1986, Gertner 1987a, 1988, Mowrer 1989).

Most stochastic functions in computer simulation studies are based on a (pseudo) random number generator which calculates numbers uniformly distributed in the interval (0,1). Thus any fraction (within the limits of computer precision) in this interval has an equal opportunity of occurring. These fractions may be used directly as probabilities, may be used to calculate normal deviates (Best 1979), or may be transformed in any other way. This process of using a random number from a uniform pseudo-random number generator is commonly termed a Monte Carlo approach, an analogy to the equal probabilities associated with an unbiased die or roulette wheel.

Many models use a single stochastic function in one component of the model, most commonly the mortality function (e.g. Moser 1972, Daniels and Burkhart 1975, Shifley *et al.* 1982). Others use a stochastic diameter increment function. Stage (1973) assumed that the only significant stochastic effect was on the diameter increment; he felt that this would have sufficient repercussions on all other functions in the model. The single stochastic component in the model of Botkin *et al.* (1972) was the number and species of trees recruited each year.

Alder *et al.* (1977) and Vanclay (1991d) suggested models in which all relationships derived from regression analyses contain a stochastic component. The difficulty with these approaches is that care must be taken to preserve the appropriate correlation between stochastic elements. For example, it is likely that the correlation between variations in diameter increment and height increment is high. However, Meldahl (1979) found that varying the correlation between predicted height and diameter increment errors had little effect on simulated stand means and distributions.

Tree List in Year N		Tree List in Year N+1	
Tree Size (cm d.b.h.)	Expansion Factor (No/ha)	Tree Size (cm d.b.h.)	Expansion Factor (No/ha)
10	100	11	15
		12	60
		13	25
20	20	22	3
		23	12
		24	5

Figure 8. Example of Record Tripling

4.8.1 Swindles

There is considerable variability in diameter increment of trees, even among trees of the same species, size and crown class, especially in TMF (e.g. Bragg and Henry 1985). This may be due to the placement of trees relative to their neighbours and to gaps in the canopy, but is a natural characteristic of these forests. The faster growing fraction of the stand is significant for the next harvest, and for the future dominant trees. Some modelling approaches (e.g. tree list and individual tree approaches) may underestimate yields if they propagate mean increments without accommodating some variability. The variability can be accommodated in a stochastic model, or by employing a ''Monte Carlo swindle'' in a deterministic model.

The term ''swindle'' has come to represent any trick which reduces the effort or improves the precision, or both, of a computer simulation (Simon 1976). Swindles are most commonly implemented in tree list models (Chapter 7) as record doubling or tripling. This enables the model to provide a deterministic prediction which emulates the average of many replications without actually carrying out the replications. Wahlenberg (1941) recognized the need to accommodate this variability and advocated that stand table projections should use actual movement data rather than movement ratios estimated from smoothed diameter increment curves.

The Prognosis model (Stage 1973, Wykoff 1986) employs a swindle in which individual tree records are incremented stochastically when there are many records, but are deterministically ''tripled'' when there are few records. In tripling (Figure 8) each tree record becomes three records with 15, 60 and 25 percent of the original expansion factor (number of trees represented by that record) and representing an increment of $\mu - 1.549\sigma$, $\mu - 0.1423\sigma$ and $\mu + 1.271\sigma$ respectively (these are derived from the normal $N(\mu, \sigma^2)$ distribution). In either case, the aggregate stand increment is assumed to be deterministic.

Vanclay (1988a, 1989a) doubled tree records, with new records having half the original expansion factor and 0.8 and 1.2 times the predicted increment in subtropical *Callitris* forest (Vanclay 1988a), and having 75 and 25 percent of the stocking with 0.9 and 1.3 times the predicted increment respectively in TMF (Vanclay 1989a). The relative increment rates for each fraction are determined from the normal distribution for Prognosis (Stage 1973), and from the distribution of increments in PSP data (Vanclay 1989a). The relative proportions in each fraction (15:60:25 in Prognosis, 50:50 in *Callitris* and 75:25 in TMF) were subjectively determined, and as yield estimates may be affected, these assumptions should be subjected to sensitivity analyses.

4.8.2 Variance Approximation

One application of stochastic simulation studies is to investigate the ''quality'' of predictions. Variance approximation provides an efficient alternative to such studies, and enables the variance of predictions to be estimated deterministically. It also enables the variance of the input data to be incorporated into the analysis.

Mowrer (1989, Mowrer and Frayer 1986) and Gertner (1987a, 1988) have demonstrated the use of first-order Taylor series variance approximations (Kempthorne and Folks 1971) to estimate the errors propagated through growth and yield projections. The general formula for error propagation is (Mowrer 1989):

$$\hat{Var}(\hat{Y}) = \sum_i^n \sum_j^n \left\{ \frac{d\hat{Y}}{dX_i} \frac{d\hat{Y}}{dX_j} S_{X_i} S_{X_j} R_{X_i X_j} + \frac{d\hat{Y}}{d\beta_i} \frac{d\hat{Y}}{d\beta_j} S_{\beta_i} S_{\beta_j} R_{\beta_i \beta_j} \right\}$$

Terms involving the variable X estimate the variance in the estimated response variable ($\hat{Y}$) from propagated errors (S_X) in the predictor variables (X) and their cross-products, with correlation coefficients R_X. Terms involving the variable β approximate fixed variance contributions (S_β) from the estimated regressor coefficients (β) with correlation coefficients R_β. These contributions may be obtained from the covariance matrix of the estimated regression coefficients, and reflect the quality of the various estimators used in the model. This Taylor series approximation does not include higher order terms or cross-correlations between model variables and coefficients. Such equations can be incorporated into the computer implementation of the model and provide concurrent estimates of the variances associated with each variable at any time during the simulation.

4.9 Conclusion

The design, construction and implementation of growth models can be complex and involve many skills. Few of us can expect to become experts in all these areas. However, a few guidelines may be adequate to ensure reasonable models are produced:

- Think about the design of the model first; try to keep it simple, and ensure that it meets your requirements.
- Understand the limitations of your data, and any implications they have for your model and analyses.
- Always plot the data and the fitted model to visually examine the quality of the fit.
- Know your own limitations, and if you don't understand something, seek help from a professional.

The keys to success in developing models are proper problem formulation, selection of important variables and model form, collection of good data (both quantity and quality), and the use of good coefficient estimation procedures and model validation techniques. Good modellers should rely more on their knowledge of silviculture and on biological principles of growth than on statistical tests when selecting models and developing algorithms.

5. Whole Stand Models

Whole stand models are those growth and yield models in which the basic units of modelling are stand parameters such as basal area, stocking, stand volume and parameters characterizing the diameter distribution. They require relatively little information to simulate the growth of a stand, but consequently yield rather general information about the future stand. This Chapter examines several such modelling approaches ranging from simple yield tables to sophisticated methods for predicting future diameter distributions.

5.1 Yield Tables

A yield table presents the anticipated yields from an even-aged stand at various ages, and is one of the oldest approaches to yield estimation. The concept was apparently first applied in the ‘‘Lung Ch’uan codes’’ by a Chinese woman some 350 years ago (Vuokila 1965). The technique as we know it today was devised in Europe in the eighteenth century. The first yield tables were published in Germany in 1787, and within a hundred years over a thousand yield tables had been published. Modern yield tables often include not only yield, but also stand height, mean diameter, number of stems, stand basal area and current and mean annual volume increments. Two classes of yield tables are distinguished, normal and variable density yield tables. Leary (1991) discussed the similarities between these, and illustrated that they do not form two discrete classes, but rather a continuum from normal yield tables, through variable density yield tables and other approaches, to individual tree growth models.

5.1.1 Normal Yield Tables

Normal yield tables refer to the ideal, fully stocked or normal forest. Data were usually derived from stem analyses and temporary plots, and were analysed using graphical techniques. As the technique developed in an era when only two variables could be readily analysed, only age and site were considered. Spurr (1952:254) and Vuokila (1965) discussed the various approaches used in Europe and North America for the construction of yield tables. The yields were generally tabulated by age and by site, but could also be presented as alignment charts (e.g. Reineke 1927). Normal yield tables may provide sound estimates of potential yields for even-aged stands similar in character to those used in developing the table, but are unsatisfactory for natural stands where there may be considerable variation in the age of the stands.

5.1.2 Variable Density Yield Tables

With the advent of more sophisticated calculation and analytical techniques, it become possible to incorporate more than two variables in yield calculations. It was quickly realized that stand density was desirable as a third variable as this enabled data from partially stocked plots to be employed and enabled the yield table to be applied to any stand. Numerous examples of variable density yield tables have been published, and recent examples include Edwards and Christie’s (1981) tables for plantations in Britain. These tables give height, stems per hectare, mean d.b.h., basal area, mean stem volume, volume per hectare, mean annual and cumulative volume production at five year intervals for a large number of species/site/management regime combinations.

This approach has also been applied to mixed stands. MacKinney *et al.* (1937) constructed a yield table for mixed loblolly pine stands using age, stand density and a composition index which was the ratio of basal area of pine to the total stand basal area. Duerr and Gevorkiantz (1938) used age of the main stand, site class, density and a merchantability class to predict yield of a uneven-aged hemlock and yellow birch stand.

5.2 Growth Tables and Formulae

5.2.1 Growth Tables

Yield tables usually require some estimate of stand age, and thus cannot be applied easily to uneven-aged stands. Growth tables attempt to overcome this limitation by tabulating growth under various stand conditions. Variables other than age used to characterize stands for growth prediction include volume, density, height, average diameter and crown class. These variables were often used in conjunction with estimates of site productivity and time since logging. Growth tables appear to have been developed in the U.S.A. about 1930 for predicting growth of ponderosa pine and selectively cut stands of other species.

Spurr (1952:265) gave several examples of tables constructed from these variables, but these approaches tended to be somewhat simplistic until the proliferation of computing equipment simplified correlation and regression analyses.

5.2.2 Growth Percent

Growth percent formulae have been used to predict the growth of individual trees, and of uniform stands by assuming that every tree in the stand behaves in a similar manner to the mean tree, at least for the period of projection. In uneven-aged forests, the formulae can also be applied to stand tables. Growth percent formulae can also be applied directly to the standing volume of a forest to estimate volume increment per hectare. Other formulae exist for predicting sustainable yields from a normal forest.

The great proliferation of these formulae attest to the inadequacy of all of them. Meyer (1953:241) compared three formulae used to predict volume increment per hectare: simple interest, compound interest and Pressler's formula, all of which yielded different results. Spurr (1952:237) considered five formulae for basal area increment of individual trees: compound interest, Pressler's, Schneider's, Jonson's and the Gevorkiantz-Hanzlik formulae. These may also be applied to diameter and volume increment of individual trees. Rudolf (1930) considered that the compound interest formula yielded the most acceptable results.

Wahlenberg (1941) presented a careful analysis of these methods, and concluded that they were unreliable at best, and could be very deceptive. Although these methods may give a reasonable estimate of short-term increment on the stand for which they were derived, it is inevitable that the percentage must decline in the longer term. Furthermore, the mean tree approach fails to give due allowance for volume losses from mortality.

5.2.3 Time of Passage

Yield estimates may be prepared from calculations indicating the time for a tree to grow through a diameter class, and this approach is known as ''time of passage''. This method has been used to determine cutting cycles for mixed forests in which inventory data are minimal, and may be used with data comprising only the ''leading desirables'' (viz. vigorous trees subjectively assessed to form the next harvest, Dawkins 1958:93) on permanent sample plots. Such data can be very difficult to analyse, and are often useless for the construction of growth models.

Osmaston (1956) and Dawkins (1958:138) discussed the calculations necessary to derive mean increments and times of passage. The length of the cutting cycle can be estimated from the time of passage and the cutting limits (Dawkins 1958:70).

This approach may not give reliable estimates of yield or duration. Mervart (1972) argued that because of the high serial correlation of diameter increments, and the great variation in increment, the method may reflect the average performance of the stand, but the harvested component may perform substantially better than the mean. Thus time of passage calculations based on stand mean increments may be expected to consistently underestimate yield and overestimate the length of the cutting cycle. However, leading

desirables comprise the faster growing component of the stand and are likely to form the bulk of the final crop. Thus time of passage calculations based on leading desirables may be more reliable. However, selection of these leading desirables is subjective and the intensity of selection may influence results.

All of these methods, growth tables, percentages and time of passage, suffer inherent weaknesses including subjective bias and inability to extrapolate for long periods or to other stands. Whilst these methods may give a rough indication of growth where data and computing resources are limited, better methods are available and should be used where sufficient resources are available.

5.3 Growth and Yield Equations

Conceptually, the distinction between tables and equations is unimportant as equations can be evaluated and presented as tables. However, the converse does not hold, and equations are a concise and convenient way to express growth and yield relationships. Equations can also accommodate more variables and can be estimated in a rigorous and repeatable way.

Growth and yield equations may be developed from data which do not identify the individual trees. Thus growth may be determined as the difference in volume (or basal area) of the plot at two remeasurements, and not directly from individual tree data. This poses problems accommodating growth on trees which have died since the last measure, especially where the measure interval is long. Beers (1962) and Marquis and Beers (1969) gave definitions which are widely accepted. Five basic components may be identified (Husch *et al.* 1982:291): V_1 and V_2 are live stand volume at measurements 1 and 2 respectively. M is mortality volume, C is cut volume and I is ingrowth volume. Growth may be defined as *net growth* $G_n = V_2 - V_1 + C$, or as *net growth of initial volume* $G_{ni} = G_n - I$. Gross growth is obtained by adding mortality (M) to net growth (G_n or G_{ni}). The measures of growth including only those trees alive at both measures are termed *survivor growth*. If they include growth on trees which have died before the remeasure, they are termed *accretion*. Estimates of survivor growth and accretion cannot readily be related without recourse to individual tree data. Alemdag (1975) gave further examples of the calculation of these components. The implication for modelling TMF is that if the growth model predicts accretion then mortality must be predicted explicitly, whereas if the model predicts survivor growth then mortality has been already taken into account.

5.3.1 Empirical Yield Equations

According to Avery and Burkhart (1983:280), MacKinney and Chaiken (1939) were the first to produce a yield equation derived by multiple regression. This was a refinement of their 1937 yield table, and employed an equation devised by Schumacher (1939) which expressed yield as a simple function of age, site index, stand density and composition. Schumacher based his equation on the hypothesis (MacKinney *et al.* 1937) that relative growth rate varies inversely with age ($\partial V/V \propto A^{-2}$), thus that

$$LogV = \beta_0 - \frac{\beta_1}{A}$$

This equation contains two parameters, one (β_0) which defines the upper asymptote (i.e. volume at end of the rotation), and one (β_1) which determines the rate of growth.

Schumacher (1939) assumed that the parameters β_i in this equation were simple linear functions in site index and stand density. The equation was subsequently used in many other models. Desirable features of this model include (Clutter 1963):

1. *The mathematical form of the variates implies relationships which agree with our biological concepts of even-aged stand development (Schumacher 1939).*
2. *The use of Log V as the dependent variable rather than V will generally be more compatible with the statistical assumptions customarily made in regression analysis (linearity, normality, additivity and homogeneity of variance).*

3. *The use of Log V as the dependent variable is a convenient way to express mathematically the interactions of the independent variables in their effect on V.*

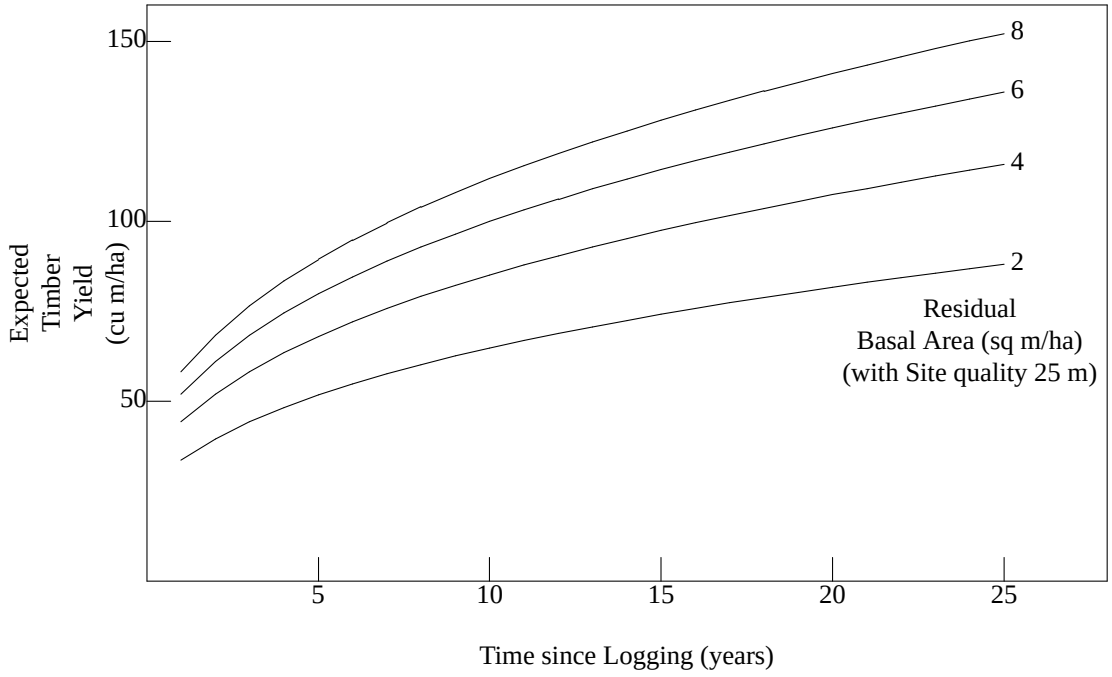

Figure 9. Yield Equation for Logged Dipterocarp Forest

Such equations have been applied rarely to mixed forests. One example was provided by Mendoza and Gumpal (1987), who predicted yield of dipterocarps in the Philippines using an empirical function employing initial basal area, site quality and time since logging (Figure 9):

$$LogY_T = 1.34 + 0.394 LogB_0 + 0.346 LogT + 0.00275 S/T$$

where Y_T is timber yield (m^3/ha, 15+ cm d.b.h.) T years after logging ($T > 0$), B_0 is residual basal area (m^2/ha) of dipterocarps (15+ cm d.b.h.) after logging and S is site quality (m) estimated as the average total height of residual dipterocarp trees (50–80 cm diameter). Whilst this equation cannot be extrapolated for long periods (or for $T \rightarrow 0$) or to other situations, it was useful for estimating the time to and volume yield of the next harvest, given the residual stand after the previous harvest.

5.3.2 Empirical Growth Equations

Yield equations have the limitation that they are static, and assume a certain management regime throughout the period of projection. Growth equations have an advantage that logging or any other treatment may be included in the projection at any time.

Buell (1945) was perhaps the first to attempt to derive a growth equation for uneven-aged mixed species stands. Buell classified his forest stands into several types according to species composition and logging history, and combined species into four groups according to their potential growth rate and stem form. He assumed that volume increment of a single tree could be expressed as a quadratic equation in diameter,

$$VI = \beta_0 + \beta_1 DBH + \beta_2 DBH^2$$

and that the volume increment of the stand was the sum of the individual increments

$$\sum VI = \beta_0 N + \beta_1 \sum DBH + \beta_2 \sum DBH^2$$

where N is stems per hectare, and DBH is diameter.

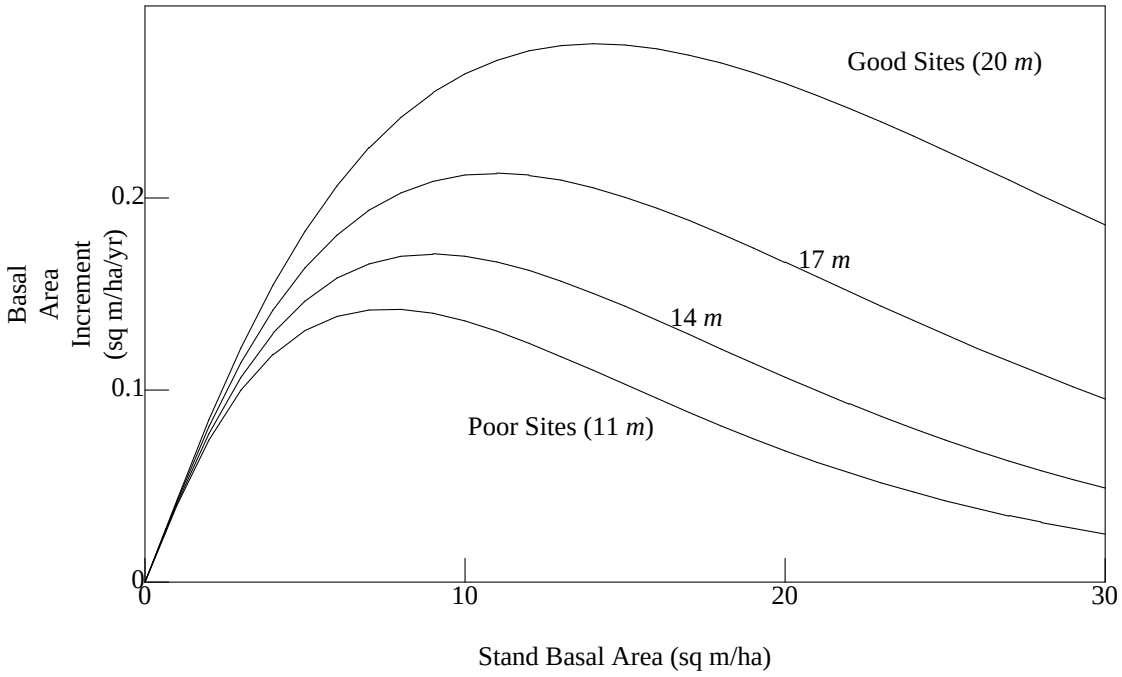

Figure 10. Stand Basal Area Increment for *Callitris* (Vanclay 1988a)

Nelson (1963) argued that stand basal area increment (BAI) of even-aged stands decreased asymptotically with age (A), increased with site index (S), and decreased as the stand basal area (B) diverged from the optimum. This led him to propose a quadratic function in basal area which also included site index and age. The statistically significant terms were:

$$BAI = \beta_0 + \beta_1 B/A + (\beta_2 + \beta_3/A + \beta_4 S)B^2$$

Nelson (1965) presented equations which could be used to predict growth or yield from mixed stands, using estimates of growth or yield respectively from pure stands of the corresponding species. His approach assumed that the yields from the mixed stands would be the average of yields from the corresponding pure stands, thus assuming that the species comprising the mixed stand grow independently without synergism or antagonism.

Vanclay (1988a) proposed a similar equation for uneven-aged *Callitris* stands:

$$Log(BAI) = -3.071 + 1.094 Log(BA) + 0.007402 BA\ SQ - 0.2258 BA$$

where BAI is stand basal area increment ($m^2/ha/ann$), BA is stand basal area (m^2/ha) and SQ is site form (m) (Vanclay and Henry 1988). This equation was deliberately formulated to provide sensible predictions for extreme values of stand basal area (Figure 10). It also seems reasonable that the better sites should have both higher increments and should reach higher stand basal areas.

Many other stand growth equations have been published. These few have been selected to illustrate the formulation of robust equations that extrapolate safely.

5.3.3 Compatible Growth and Yield Equations

Early analyses of yield failed to exploit the relationship existing between growth and yield and this led to instances where estimates from yield tables could differ from those derived from summing successive estimates from growth equations based on the same data. Buckman (1962) and Clutter (1963) argued the need for compatibility, and constructed compatible growth and yield equations which would give consistent estimates.

Buckman (1962) expressed basal area increment as a simple function of stand basal area, age and site index. He suggested that in managed stands where regular logging kept the basal area within fairly broad limits, site index and basal area could be assumed constant, and the yield could be determined from the integral of the resulting quadratic equation in age.

Clutter (1963) recognized the importance of the change in basal area with time. He used a variation of the equation proposed by Schumacher (1939):

$$LogV = \beta_0 + \beta_1 S + \beta_2 LogB + \beta_3 / A$$

where V and B are stand volume and basal area at age A years respectively, and S is site index. This can be differentiated to give (amongst other possibilities) the growth equation

$$\frac{dV}{dt} = \beta_2 \frac{V}{B}\left(\frac{dB}{dt}\right) - \beta_3 \frac{V}{A^2}$$

Clutter expressed basal area as a differentiable yield function in age and site index, so that basal area increment (dB/dt) could be estimated from stand basal area, age and site index. This led to five compatible equations which could be used to derive estimates of present standing volume, basal area increment, volume growth, projected future basal area and projected volume yield. Sullivan and Clutter (1972) refined this equation to provide numerically consistent estimates of growth and yield from initial stand age, basal area, site index and future age. Burkhart and Sprinz (1984) presented a method for simultaneously estimating the parameters in the Sullivan-Clutter approach.

5.3.4 Theoretical Equations

The Bertalanffy (or Chapman-Richards) and several similar equations (e.g. Zeide 1989, 1990) can be expressed as growth or yield equations in the same manner as the compatible growth and yield equations discussed above. However, they are sometimes thought to be superior to empirical equations as they are supposedly biologically based.

By expressing stand volume as an allometric function of basal area ($V = \beta_0 B^{\beta_1}$), Moser and Hall (1969) were able to express volume growth of a mixed forest as a function of basal area. They predicted basal area increment from stand basal area using the Bertalanffy equation ($BAI = \beta_2 B^{\beta_3} - \beta_4 B$) and thus could express volume increment:

$$VI = \beta_1 V(\beta_2 B^{\beta_3 - 1} - \beta_4)$$

The integral form of this equation may be obtained by integrating the basal area increment function, and substituting for volume. Thus the cumulative volume growth over any specified period of time can be estimated. This approach leads to a compatible growth and yield equations for both basal area and volume. Although it takes no account of site, this equation has been used in many growth and yield studies in uneven-aged stands, including *Pinus taeda* - *P. echinata* (Murphy and Farrar 1982) and pinyon-juniper (Chojnacky 1988).

Despite this general acceptance, it is not really a theoretical equation because of a flaw in its basic formulation. The method assumes that the Bertalanffy equation holds when it is applied to populations (i.e. the forest stand) rather than to individual organisms. This assumption is flawed as catabolism is not proportional to the stand volume or basal area (since heartwood does not respire), and anabolism is asymptotic to an upper limit (reached at canopy closure). Several empirical studies have reported negative coefficients when this equation is fitted at the stand level (e.g. Moser and Hall 1969, Murphy and Farrar 1982, Chojnacky 1988), while theory suggests that positive coefficients should hold.

Consider the following argument. Assume that living tissue may be proportional to girth times height or approximately $\sum D^{1.5} \approx B^{0.75}$ (where B is stand basal area). Expressing stand volume increment as the difference between gross stand anabolism (A) and stand catabolism (bB^c, $c \approx 0.75$), put $dV/dt = A - bB^c$ and assuming that an allometric relationship exists between stand volume and basal area ($V = eB^f$, $f \approx 1.0$),

$$\frac{dB}{dt} = \frac{A}{ef} B^{1-f} - \frac{b}{ef} B^{c+1-f}$$

Anabolism is related to the amount of sunlight intercepted by the photosynthetic surfaces of the trees in the stand, and may exhibit an allometric relationship for low basal areas, but should be asymptotic as basal area increases, and a constant may be sufficient for well stocked stands. For an allometric relationship $A = gB^h$ with h small ($h \to 0$),

$$\frac{dB}{dt} = \frac{g}{ef} B^{1-f+h} - \frac{b}{ef} B^{c+1-f} \approx pB^h - qB^{0.75}$$

This argument suggests that there is no biological basis to support the use of the Bertalanffy equation for modelling stand basal area increment. However, the equation continues to provide a good empirical model for compatible growth and yield estimates. Leech (S.A. Woods & Forests, *pers. comm.*) has fitted the equation $dY/dt = nY^m - pY^v$ to plantation growth data. He found that the parameters m and v were highly correlated (This causes problems in fitting the model to the data, see Section 4.5.3), and that the usual Bertalanffy equation (with $v = 1$) was a more convenient form that provided a fit almost as good. The real problem is that additional state variables are required, and top height should be useful for even-aged stands (Garcia 1990).

Garcia (1974) examined an alternative theoretical equation for the volume growth of plantations. He assumed that (1) gross increment in stem volume is proportional to the increase in dry weight $\Delta V \propto \Delta M$, (2) stand density remains sufficiently high to ensure full occupancy of the site and that assimilation per hectare is constant, (3) respiration is proportional to the quantity of live tissue which was in turn proportional to the surface area of the stems and thus the product of mean diameter (D) times the height (H) and the number of stems (N). Thus Garcia proposed that stem volume increment (ΔV) could be predicted as

$$\Delta V = a - bNHD$$

Assuming a constant form factor (f) so that $V = fBH$ with stand basal area is $B = \pi ND^2/4$,

$$D = \sqrt{\frac{4}{\pi f}} \cdot \sqrt{\frac{V}{NH}}$$

and thus

$$\Delta V = a - c\sqrt{VNH}$$

Garcia (*pers. comm.*) later found that although plausible, this model did not fit the data well and that a more flexible model was required. His demonstration does however serve to indicate some weaknesses of the Bertalanffy equation when applied to forest stands.

5.4 Systems of Equations

A better understanding of growth in uneven-aged stands may be obtained if components of growth are individually identified and expressed collectively as a system of equations to predict stand growth.

Furnival and Wilson (1971) developed a growth and yield model as a system of equations and solved all the coefficients using simultaneous estimation. They formulated their yield model for white pine stands as the following system:

$$\begin{aligned} LogH &= a + bLogA \\ LogN &= c + dLogD \\ LogB &= e + fLogH \\ LogF &= g + hLogD \\ LogB &\equiv LogK + LogN + 2LogD \\ LogV &= LogF + LogH + LogB \end{aligned} \qquad (5.6)$$

where V is volume (m^3/ha), H is average stand height, A is average main stand age at breast height, B is basal area (m^2/ha), D is d.b.h. of tree of mean basal area, N is number of trees per hectare, F is the cylindrical form factor, and K is a constant ($\pi/40000$). Equation (5.2) is Reineke's (1933) stand density equation, and equation (5.5) follows from the definition of basal area. Equation (5.3) was used by Schumacher and Coile (1960) in their growth and yield model. Equation (5.6) is the identity defining

volume, and equation (5.4) estimates the form factor for the stand. Equation (5.1) was a convenient function suggested by the data (Furnival and Wilson 1971).

To derive a system of growth (c.f. yield above) equations, Furnival and Wilson (1971) differenced (not differentiated) the equations (5.1) to (5.3). When they solved these systems for both growth and yield, different coefficients were determined. Furnival and Wilson attributed this to deficiencies in the model, and suggested that the growth model was the more reliable approach. They used three techniques, all commonly used in econometrics, to solve the systems: direct least squares, two stage least squares, and three stage least squares; and found that three stage least squares used in conjunction with the growth model formulation of the system gave the smallest standard errors of estimation. They argued that the advantage of systems formulation for a growth model is that a variance-covariance matrix is produced, and can be used to estimate the standard errors of predicted growth and yield. However, there are several problems with this system. Equation 5.5 is an identity, and 5.6 is effectively exact, but equations 5.1 to 5.4 are approximations which are not realistic. Equation 5.2 accommodates only mortality due to limiting competition, and fails to account for other sources of mortality.

Moser (1972) identified seven components of a mixed hardwood stand in Wisconsin as Y_1 the number of trees (greater than 18 cm d.b.h.) at a given instant, Y_2 the basal area of these trees, Y_3 the number of trees dying during the period (t_0, t_n), Y_4 the number of recruits during the period, Y_5 the basal area of the dying trees, Y_6 the basal area of the recruits, and Y_7 the cumulative basal area growth of the surviving original trees (Y_1) during the period (t_0, t_n). The intuitive relationships

$$Y_1 = Y_1(t_0) - Y_3 + Y_4$$
$$Y_2 = Y_2(t_0) - Y_5 + Y_6 + Y_7$$

may be differentiated to give the rate of change in numbers and basal area, or the growth of the stand. Thus the system of equations may be built up as

$$\frac{dY_1}{dt} = \frac{-dY_3}{dt} + \frac{dY_4}{dt}$$

$$\frac{dY_2}{dt} = \frac{-dY_5}{dt} + \frac{dY_6}{dt} + \frac{dY_7}{dt}$$

Moser (1972) used the derivative form of the simple death process $dY_3/dt = \beta_0 Y_1$, which assumes that mortality is proportional to the number of trees in the stand. He assumed that the size of the dying trees varied according to the negative exponential distribution, and developed the stochastic function

$$\frac{dY_5}{dt} = \beta_1 Y_1(\beta_2 LogX + \beta_3)^2$$

where X is a random variable in the interval $(0, 1)$, to predict the basal area dying. He expressed recruitment as a function of the quadratic mean diameter (diameter of tree of mean basal area) $dY_4/dt = \beta_4 e^{hY_1/Y_2}$, and the basal area recruited Y_6 as a constant times the number recruited. Surviving basal area growth was predicted using the Bertalanffy equation fitted in a previous study (Moser and Hall 1969).

These equations could not be fitted simultaneously as in Furnival and Wilson's (1971) study because of the stochastic element, but were fitted separately, and thus a variance-covariance matrix could not be produced. The system of equations can be reformulated into two equations to predict stocking and basal area growth. The stocking equation is a simple deterministic equation, and the basal area growth equation is stochastic. Although the system gave adequate estimates of growth, there was considerable bias evident in the mortality functions. Moser (1972) suggested that the simple death process should be replaced with a stochastic function.

Shifley *et al.* (1982) used a similar system of equations to predict growth and yield of elm-ash-cottonwood forests in Indiana. They used similar functions to predict number and basal area of recruits, but altered the other functions slightly. Survivor basal area growth was assumed to be directly proportional to the stand sum of diameters, and the recruited numbers of trees was expressed as an exponential function in stocking and basal area. For stands of exceptionally low or high basal area, the mean size of dying trees was assumed to be the same as the survivor tree of mean basal area. Elsewhere, the mean diameter of dying trees was predicted using the inverse of the cumulative form of the Weibull frequency distribution:

$$D_m = \beta_0 + \beta_1(-Log(1-X))^{\beta_2}$$

Basal area dying was then computed as the tree basal area corresponding to the predicted diameter, multiplied by the number dying. Shifley *et al.* (1982) cautioned that this method could not be expected to perform well for stands which differed greatly from those for which these relationships were developed. Tests of the model showed that the mean of fifty iterations could give good estimates of future basal areas and volumes. One 15 year simulation produced estimates of volume to within 3 percent of the actual volume.

5.5 State Space Models

Garcia (1979, 1983, 1984, 1988) applied the state space approach to modelling plantations. The stand is represented by a small number of state variables, usually stand basal area, number of trees per hectare and top height. It is assumed that these state variables summarize the historical events affecting the future development of the stand, and thus that future states can be determined by the current state and future actions, and that other variables of interest, such as volume can be derived from these state variables (Goulding and Shirley 1979).

The uni-variate Bertalanffy equation can be expressed as a linear differential equation with a power transformation of the state variable. For example, for site index curves,

$$dH^c/dt = aH^c + b$$

where H is the state variable top height and a, b and c are parameters to be estimated. Garcia (1979, 1984, 1988) used the multi-variate generalization:

$$d\underline{X}^{\mathbf{c}}/dt = \mathbf{a}\underline{X}^{\mathbf{c}} + \underline{b}$$

with $\underline{X}^{\mathbf{c}}$ defined as

$$\underline{X}^{\mathbf{c}} = e^{\mathbf{c}Log\underline{X}}$$

where $\underline{X}$ is an n-dimensional state vector and **a**, **c** and $\underline{b}$ are n-dimensional matrices and vectors of parameters. Garcia (1984) found that **c** was independent of site index, and that **a** and $\underline{b}$ changed by a constant factor with site index. Thus site index acted like a change in the time scale, and could be accommodated in the model by putting $\tau = bt$ in place of t. However, this response with site index may not apply to all localities (Garcia 1988). These parameters were estimated using maximum likelihood estimation using a general purpose optimization procedure (Garcia 1984, 1988). Although this approach gave excellent predictions and provided an effective framework for a series of plantation growth models, Garcia (1988) cautioned that ''these methods cannot be recommended for general use by the casual growth modeller; the capacity to understand the techniques, and modify them if necessary, is essential''. Adapting the approach for TMF would introduce many additional complexities, as it is unrealistic to assume that TMF could be described adequately with only three state variables.

5.6 Whole Stand Transition Matrices

Markov chains provide a concise way to summarize the behaviour of a system, and have been used for modelling forest stands. Consider a hypothetical system S, which has n distinct states $S_1, S_2, \cdots, S_n$. If the system starts in state S_i, then in a single time interval, it has probability P_{ij} of moving to state S_j. Provided that these P_{ij} depend only on the current state S_i and not on the past history, these probabilities can be expressed in a square matrix, termed the transition probability matrix or stationary Markov chain. This is a very concise way of expressing the behaviour of a system, but reveals little of the reasons for the change of state. Two advantages of the approach include the ease of determining the state of the system at any multiple of the time interval (by multiplying the matrix by itself), and that the steady state of the system may be readily determined by repeatedly squaring the matrix (Waggoner and Stephens 1970).

Hool (1966) adopted this approach to predict the behaviour of a mixed species even-aged forest under certain management regimes. He introduced dynamic programming by defining activities A_k (e.g. thinning), which take the system from the state S_i to state S_j with probability $P_{ij}A_k$. With each activity

induced transition there is an associated revenue $R_{ij}A_k$, and each state S_i has an associated value V_i. This approach enables optimum thinning schedules to be determined. Hool used a two year transition interval, and identified 36 states on the basis of logging history (thinned/unthinned), standing volume (6 volume classes each 40 cu m) and stocking (3 classes each 170 stems/ha). He considered three activities, undisturbed growth (over the two year transition interval), thinning and harvesting. Hool did not reveal how he developed his transition matrices, which are not published. His discussion is concerned largely with applications of the approach, and recommendations for management of forests in Indiana. He showed that the mean transition time in any state is 2.4 to 3.8 years, depending on the state, and that if left undisturbed, the woodlands will remain in states 14, 16 or 17 (about 200 cu m/ha with 180 stems/ha) with probability 0.86.

Binkley (1980) used whole stand transition matrices to examine succession in forest stands. The states of his matrix indicated the dominant species on each plot. His analysis suggested that assumptions inherent in the method were untenable (see section 6.3), and that transition matrices were not a reliable way to predict forest stand dynamics.

5.7 Whole Stand Distribution Models

The approaches discussed above provide rather limited information about the forest stand, in most cases only the predicted volume. This is only one of the estimates required by the forest manager. Effective management also requires information about sizes and species contributing to this stand volume.

An early solution to this problem was to include stand or stock tables in published yield tables. Many yield tables which were presented as alignment curves also included axes giving the average and minimum stem sizes as well as various other information regarding the dimensions of the crop. A remarkable example of this is Reineke's (1927) ''composite chart'' which concisely expresses dominant height and gives average d.b.h., basal area, stocking and total volume under bark for three stand fractions (dominants only, 10 cm plus, and 17 cm plus) at any age (1 to 100 years).

The technique of using an equation to predict diameter distributions directly from the stand variables appears to have been proposed by Bennett and Clutter (1968) who used age, site index and stocking to quantify the maximum and minimum stand diameters, and the distribution of diameters between these extremes, using the transformed beta probability density function (p.d.f.). An independent test (Burkhart 1971) revealed that for a range of stand conditions, the technique could predict plot volume to within five percent of the true volume in 18 percent of the cases, and only six percent of cases differed from the true volume by more than 50 percent. The test plots used varied in area, but contained 64 original planting positions.

Bailey and Dell (1973) and Schreuder and Swank (1974) demonstrated that diameter distributions in even-aged forest stands can be quantified by the two parameter Weibull distribution. Since height and volume can be expressed as an allometric function of diameter, they are distributed according to the Weibull distribution whose parameters are given by a simple transformation (Stacey and Mihram 1965). Hyink and Moser (1979) applied this technique to uneven-aged forest. They noted that given the total number of trees, a diameter distribution, and some function $g(x)$ of diameter, the definite integral of the product of the function $g(x)$ and the distribution could predict stand characteristics corresponding to the function $g(x)$ for any component of the stand. Thus the diameter distribution could provide estimates of top height, mean height, total volume and merchantable volume removed in logging. They assumed that the distribution could be adequately represented by the three parameter Weibull distribution, and that the growth of the stand could be accounted for by changing these parameters. The Weibull (1951) distribution is characterized by three parameters. Hyink and Moser (1979) assumed that the location parameter (a) was a constant 17 cm d.b.h., the lower limit of measurement; leaving the scale (b) and shape (c) parameters and the number of trees to be determined. They predicted these with empirical functions:

$$
\begin{aligned}
dI/dt &= f(N, \textstyle\sum D) \\
dM/dt &= f(N) \\
db/dt &= f(N, dI/dt, \textstyle\sum D, b) \\
dc/dt &= f(N, dI/dt, \textstyle\sum D_{<21}, b, c)
\end{aligned}
$$

where N is stems per hectare, I is recruitment at 17 cm d.b.h., M is mortality, $\sum D$ is the sum of diameters,

$\sum D_{<21}$ is the sum of diameters less than 21 cm, and *b* and *c* are the scale and shape parameters of the Weibull distribution respectively. This approach makes more sense if the reader realizes that the parameter *b* is related to the stand median diameter, and *c* approximately corresponds to the variance of the diameter distribution (Clutter and Allison 1974). Lynch and Moser (1986) and Bowling *et al.* (1989) modified the method for mixed stands by employing several sets of equations, one set for each species. This approach is known as the parameter prediction approach (Reynolds *et al.* 1988).

An alternative is the parameter recovery approach which has been found to give better predictions (Reynolds *et al.* 1988). Instead of predicting the three Weibull parameters directly, the stand basal area and mean diameter (arithmetic, not quadratic) can be predicted, and the distribution determined by matching the moments of the Weibull to the predicted stand attributes.

The utility of both methods depends on the ability of the Weibull p.d.f. to adequately characterize the diameter distribution. Although the Weibull p.d.f. can describe a great variety of shapes (e.g. Krug *et al.* 1984), it is always uni-modal (or L-shaped), and thus cannot cope with stands which, for example, exhibit a distinct two age structure. In many natural unlogged stands the Weibull may perform well, but logged stands may pose a problem. Certain types of selection logging may not distort the diameter distribution greatly, but diameter limit cutting will often produce stands which cannot be accommodated by the Weibull distribution. Multi-modal stands could be accommodated by using a series of Weibull functions but this becomes rather complex, and it may be better to adopt a stand class or tree list approach for such stands.

5.8 Conclusion

Whilst whole stand models have been demonstrably useful for plantation modelling, they appear of limited utility for TMF, where the large number of species and the potential for multi-modal size distributions creates difficulties in characterizing the stand with few stand-level variables.

6. Stand Class Models

Stand Class models employ a class of trees as the basic unit for modelling. One common stand class approach is stand table projection which divides the forest stand into a number of equally sized d.b.h. classes. The stand class approach is a compromise between whole stand models and individual tree models. Whole stand models may be considered stand class models in which a single class encompasses the whole stand. Individual tree models may be considered to have a class for each individual tree. Thus the stand class approach encompasses those models which divide the stand into two or more classes, but with fewer classes than the total number of trees. The model may consider each class more or less independently of other classes, or it may account for the other classes explicitly or implicitly.

However, the stand need not be partitioned into standard classes: it may be broken into ''cohorts'' or groups of trees with similar characteristics (e.g. species and size). Clutter and Allison (1974) used twenty five cohorts, each with equal frequency (four percent of the trees) to model *P. radiata* plantations. Alder (1979) and Alder and Schneider (1979) used deciles. Tree list approaches (e.g. Stage 1973, Leary 1979) are more flexible and enable cohorts containing unequal numbers of trees. Diameter is not the only criterion that may be used to divide forest stands into classes or cohorts. Monserud and Ek (1977) used height classes to predict the development of uneven-aged hardwoods.

Many stand class approaches originate from the classical method of stand table projection. Transition matrices are a logical and formal extension of this approach. Equations improve on this method by taking some account of stocking in classes other than the subject class. The mean tree approach uses a number of trees each representing a cohort. This is a logical extension of the whole stand approach of a single mean tree sometimes used to predict the growth of even-aged stands. Finally, the stand class distribution models extrapolate from the mean tree in each class by assuming some distribution of stems within or between classes. Tree list models (e.g. Stage 1973, Leary 1979) use the mean tree (cohort) approach with class sizes approaching individual trees.

6.1 Stand Table Projection

Stand table projection is one of the oldest techniques used to determine the future composition of uneven-aged forests. The method predicts the future stand table from the present stand table using estimated diameter increments for each class. Estimates of diameter increment may be derived from several sources; increment of the coming or past period (Carron 1968:148), from growth percent, derived from Herrick's (1938) formula or Chapman's (1942) method, or from increment equations prepared using linear regression analyses.

Three methods can be used to forecast the future stand (Table 4). The first assumes that all trees in each diameter class are located at the class midpoint, and that all trees will grow at the same average rate (e.g. Husch *et al.* 1982:296). The method essentially involves projecting the class boundaries so that future classes contain the same trees (if there is no mortality), but may have different class boundaries (and widths), which may be inconvenient for some applications. It also takes no account of the variable nature of diameter increment.

The second method assumes that trees in each diameter class are uniformly distributed through the class and that each tree grows at the average rate (e.g. Husch *et al.* 1982:296). For each class, a ''movement ratio'' is determined from the class width and average increment, and this indicates the proportion of trees moving to the next class. For projections involving stands with a finite number of trees, some decision must be made regarding rounding fractions of trees (e.g. Carron 1968:167). In projections for large forest areas where fractions may be considered acceptable, the method may lead to bias in predicted growth of the largest stems, as the distribution in the largest class is rarely uniform. Both these methods ignore dispersion of individual increments.

The third method, devised by Wahlenberg (1941) attempts to account for the variation in growth rate within

TABLE 4. Three approaches of Stand Table Projection

Projection Method	Size Classes (cm diameter)					Mean
	10–19	20–29	30–39	40–49	50–59	Diameter
Initial Stand	40	30	20			22.8
Stand after 1 projection cycle						
Class Boundary Method†		40	30	20		31.8
Usual Method (90% grow 1 class)	4	39	29	18		31.8
Wahlenberg's Method (20% grow 2 classes, 50% 1 class)	12	29	29	16	4	31.8

†For this method, new size classes are actually 19–28, 29–38, 39–48 cm d.b.h.

any diameter class. This is achieved by using the actual movement of trees rather than movement ratios. An example given by Husch *et al.* (1982:299) illustrates one instance where the movement ratio approach predicts that 94 percent of trees will move one class and 6 percent remain, whilst Wahlenberg's method predicts that 20 percent will move two classes, 50 percent will move one class, and 30 percent remain. Wahlenberg's method may provide more reliable estimates of timber yields.

One problem with the last two approaches to stand table projection is the proliferation of classes with fractional numbers of stems: this difficulty is avoided by the first approach. The impact of this proliferation can be reduced by using a longer projection interval, employing more and narrower classes, or by assuming a non-uniform distribution of stems within each class. The problem may also be overcome by accumulating the movement probabilities for classes with few stems until a reasonable number of stems can be projected. Conversely, the first approach fails to indicate the variability of increment in the projected stand, and may underestimate yields for the harvested component of the stand.

6.2 Transition Matrices

Transition matrices are a logical and formalized extension of the method of stand table projection. Three variations of this approach exist, and differ in their underlying assumptions.

6.2.1 Markov Chains

Markov chains (Chains and matrices are more or less synonymous, but chains refers to the principle, matrices to the table of values) have been extensively used in statistics and computer science to analyse the behaviour of systems, and have also been used in the biological sciences. Markov chains assume that at any time a system can be in any one of a finite number of states, and during the next time interval, has a known probability of moving to any other state, dependent only upon the current state.

In forestry terms, one interpretation is that a tree is considered to be in one of the diameter classes of the stand table. During the next period, it must either remain in the class, grow into another class, be felled, or die. We know that it is most likely to remain in the same class or to grow into the next class; there is a small probability that it may die or be felled, and it is exceedingly unlikely to do anything else.

This leads to the apparent contradiction that although a Markov chain is a stochastic process, it is generally used as a deterministic yield model. A Markov chain could be used as a stochastic model if it were applied to individual trees, using random probabilities, but when applied to populations of trees in a deterministic manner, it obviously yields deterministic results. Thus Markov chains could provide a basis for a compatible deterministic/stochastic growth model.

A Markov chain contains two types of states, transient and absorbing. Transient states are those in which the system spends a finite duration: it must eventually leave these states. In the present application, the diameter classes are the transient states, as every tree must eventually grow into the next class, or be logged

or die. Absorbing states are those from which the system can never leave. In our case, death and harvesting represent absorbing states.

The probabilities of movement are generally expressed as a matrix (**M**), and can be used to predict change during a single time interval:

$$\underline{V}_1 = \mathbf{M} \bullet \underline{V}_0$$

or over a number of time intervals:

$$\underline{V}_n = \mathbf{M}^n \bullet \underline{V}_0$$

where **M** represents the Markov matrix containing the probabilities of movement, and $\underline{V}_0$ and $\underline{V}_n$ are vectors describing the initial and final states respectively. In forestry, $\underline{V}_0$ is generally a list representing the initial numbers of trees in each diameter class.

In order for these expressions to hold, two assumptions must be made. The first, termed the *Markov assumption*, requires that the probability of any event must depend only on the initial state. Thus the probability that a tree grows into the next class depends only upon the class that the tree is presently in, and not upon the characteristics of that tree (except those that define the class), upon any other tree, or on the numbers of trees in any class (or in the stand as a whole). The second assumption, the *stationary assumption*, requires that these probabilities do not change over time. These Markov and stationary assumptions may be quite restrictive in forestry applications. The Markov assumption means that there is no way to account for suppression of smaller trees by the presence of larger trees. The stationary assumption means that a Markov matrix cannot account for increasing stand basal area and/or stand height. Theoretical (e.g. Hulst 1979) and empirical (e.g. Binkley 1980, Roberts and Hruska 1986) studies suggest that these assumptions are untenable for modelling forest dynamics.

The stationary assumption also requires that a matrix incorporating harvesting must represent the same time interval as the cutting cycle, otherwise the matrix must represent the growth of the stand in the absence of logging. This problem can be overcome by constructing a matrix ($\mathbf{M}_5$) representing growth over a five year period in the absence of logging, and a matrix (**L**) representing an instantaneous time interval, containing the logging rules (e.g. Buongiorno and Michie 1980). Then, a thirty year cutting cycle could be given as

$$\underline{V}_{30} = \mathbf{M}_5^{\,6} \bullet \mathbf{L} \bullet \underline{V}_0$$

Despite these restrictive assumptions associated with Markov chains, a considerable number of publications attest to their usefulness in forestry. Bruner and Moser (1973) and Moser (1978) discussed a study which used 25 states: 22 one inch (2.5 cm) d.b.h. classes from 8 inches (20 cm) to 29 inches, one class for stems larger than 29 inches (74 cm), and one class each for mortality and harvesting. Both these studies considered matrices which were used to post-multiply the state vector, thus their matrices represent the transpose of matrices for the more common pre-multiplication approach. Because Bruner and Moser had only one absorbing state each for harvesting and mortality, they could only predict the numbers of dead and cut trees, not the sizes of these trees. This problem can be overcome by adopting more states, one for each dead or cut size class required (e.g. Rorres 1978, Peden *et al.* 1973). Cassell and Moser (1974) compiled six matrices according to the shade tolerance classes of the species comprising the stand. Bragg and Henry (1985) described a matrix model which was used to predict timber yields for managed TMF in Queensland.

Advantages of the Markov chain approach are that it is easy to apply, and that it can utilize PSP data in which the individual trees were not identified. Where individual trees are not identified, several methods exist for extracting the necessary probabilities of movement (e.g. Carron 1968:148). Logistic regression may also be used to derive movement probabilities if individual trees are identified. This method is efficient and may be useful where few data are available.

The Markov chain approach may give reliable results. Accuracy of prediction seems to depend upon the number of time intervals, rather than the total time *per se* (Bruner and Moser 1973). It appears that stands may be projected for several intervals before departure from reality becomes significant (Enright and Ogden 1979). Predictions become unreliable if stand conditions (e.g. stand basal area) depart greatly from those conditions prevailing in the data from which the matrix was developed (i.e. violates the stationary assumption). Whilst stand conditions remain within a narrow range similar to the development data, the stationary assumption may not be violated. Thus the method may give better results for undisturbed near-climax stands than for successional stands following logging or other disturbance.

The method has a number of other disadvantages. The stationary and Markov assumptions may not be

acceptable for many forest stands (Hulst 1979, Roberts and Hruska 1986). Matrix approaches imply an exponential increase in numbers, whereas populations usually approach asymptotically an upper limit. However, by including additional parameters, more flexible patterns of population increase can be modelled (e.g. Usher 1971). Where cut and dead stem sizes are recorded, these usually represent the size at the start of the projection period. This may be a reasonable estimate for dead stems, but is likely to underestimate size for cut stems, particularly if the projection interval is long. However, Rorres (1978) has demonstrated that the size of a tree at the time of harvest or death can be predicted with some additional complexity. Another disadvantage is that generally projections can only be made in multiples of the measurement interval for the PSPs. However, a one-year matrix (**A**) may be estimated from an *n*-year matrix (**P**) such that $\mathbf{A}^n \approx \mathbf{P}$ (Harrison and Michie 1985). An exact solution cannot be found. The one-year matrix must have entries on at least two diagonals (i.e. bi-diagonal or Usher matrix) or no growth can occur, and this enables some trees to move *n* classes in *n* years, even though the original matrix (**P**) may predict less movement.

A further disadvantage of Markov matrices is the number of parameters required and the implications on the probable variance of each parameter. Moser's 25 state matrix requires the estimation and processing of 625 constants. Admittedly, many of these are zero, but the size of the matrix cannot be reduced. Nor can the matrix accommodate differences in site productivity.

6.2.2 Leslie and Usher Matrices

Usher matrices attempt to overcome the size problem of Markov matrices. By choosing the time interval and class width so that no tree can grow more than one class during the period (for convenience I shall call this the *Usher assumption*), a substantial saving in space can be made. With a true Markov matrix, the columns should sum to unity (assuming mortality is included), and this means that the regeneration (or ingrowth) cannot be predicted. However, by relaxing this restriction, recruitment can be predicted by employing non-zero values in the top line of the matrix. These values are not probabilities, but reflect fecundity, the number of offspring for each individual in the corresponding cell of the state vector.

Fecundity values in the matrix allow the amount of recruitment appearing to vary according to the presence of trees in various classes, and it is even possible to construct a matrix which recruits regeneration only on the death of a larger tree, but complicated to have density-dependent regeneration. Numerous studies have investigated the sensitivity of the eigenvalues and vectors to noise in the matrix. Usher (1966) found that small variations in the fecundity terms of the matrix had very little influence on the eigenvalues and vectors. He also showed that estimates of growth have greater influence on the eigenvalues and vectors than the estimates of fecundity.

Growth can be accounted for with a single probability (either a tree moves into the next class, or it does not), and the matrix may be reduced to four vectors: a growth vector, a fecundity (recruitment) vector, a harvesting and a mortality vector. Thus for Moser's (1978) 25 state system, the number of constants to estimate and process could be reduced from 625 to 75 (Moser did not consider recruitment). Typical applications may contain 20–25 states and require the estimation of 80–100 parameters, more than is required in many equation approaches. The square, or any higher power, of a Usher matrix yields a Markov matrix.

The simplifying Usher assumption does not introduce bias as does the movement ratio approximation of Wahlenberg's method for stand table projection. The movement ratio assumes that all trees will grow at the same rate, and allows them to move more than one class. Usher matrices specify that the time period of projection must be sufficiently short that a tree cannot grow more than one class.

Leslie (1945, 1948) pioneered the use of these matrices for animal populations where the classes represented ages. The technique was adapted for stages of insect development by Lefkovitch (1965), and for forestry (diameter classes) by Usher (1966). A rare application of a Leslie matrix to forest stands was Bosch's (1971) study of redwoods, which used age classes and in which regeneration appeared only on the death of another tree. Most studies utilize the diameter class matrix advocated by Usher (1966), but are still commonly termed Leslie matrices. However, there is an important distinction between Leslie and Usher matrices. With a Leslie matrix of age classes, all surviving individuals progress (''age'') to the next class each cycle. With a Usher matrix, only a proportion of surviving trees grow into the next class, whilst those with little or no growth remain in the same class.

Usher matrices, perhaps because of the efficiencies inherent in the Usher assumption, have been more widely used than Markov matrices. Usher (1966) argued that the largest latent root (dominant positive eigenvalue) of the matrix indicated the maximum exploitation, and that its latent vector (eigenvector) indicated the stable stand structure. Usher (1976) derived estimates of optimum yield and rotation length for *P. sylvestris* plantations. Rorres (1978) continued this analysis to prove that the optimal sustained yield harvesting regime is a cutting limit regime which removes all the stems in only one class, removes a proportion of the stems in several smaller classes, and leaves all the remaining smallest classes untouched. This is consistent with some selection harvesting guidelines but at odds with harvesting practices in many tropical countries.

Plant demographers attach some importance to the eigenvalue. The significance of the eigenvector and value is that the eigenvector corresponds to the theoretically stable population structure; if such a population is projected for another period, the proportion in each class will remain the same, but the actual number will vary according to the eigenvalue. The eigenvalue thus indicates whether the population is increasing or decreasing in number. In theory, pioneer and light demanding plants in undisturbed forest should have eigenvalues less than unity, whilst shade tolerant plants should have values greater than 1.0. Values greater than one for pioneer species would indicate continuing disturbance. Hartshorn (1975) obtained a value of 1.002 for the shade tolerant climax species *Pentaclethra macroloba* and Enright and Ogden (1979) observed values of 1.0202 and 1.0115 for the shade tolerant *Araucaria cunninghamii*, and values from 0.9889 to 1.0884 for the shade intolerant mid-successional *A. huntsteinii*. It is not clear whether the eigenvalues are a characteristic of the species, a characteristic of the present state of the forest, or an artifact of the method. Eigenvalues assume an exponential increase in the number of trees in each size class. Thus the eigenvector may indicate optimal stand structure, but not stand density. The problem of determining the optimum level of growing stock can not be resolved with eigenvalues.

6.2.3 General Matrices

The Markov assumption that the probability of change of state depends only on the initial state, is a severe restriction to impose, particularly in dealing with uneven-aged forests. If this assumption is discarded, it is possible to construct a matrix model in which recruitment is density dependent, or which accounts for suppression of smaller trees by introducing negative values in the upper right triangle of the matrix. One advantage of the Markov chain is that it may be presented graphically as a network or tree diagram, it is easy to conceptualize, and easy to locate errors. It is not clear how one should represent a negative probability! If attempts are made to calculate eigenvectors, the state vector must be inspected after each iteration to ensure it contains no negative values.

$$\begin{pmatrix} 130.8 \\ 65.0 \\ 42.4 \\ 31.5 \\ 22.2 \\ 14.1 \\ 6.2 \end{pmatrix} = \begin{pmatrix} 0.82 & -.04 & -.21 & -.43 & -.68 & -.98 & -1.31 \\ 0.23 & 0.70 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.26 & 0.67 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.30 & 0.65 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.30 & 0.66 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.30 & 0.81 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.19 & 0.86 \end{pmatrix} \times \begin{pmatrix} 100 \\ 60 \\ 40 \\ 30 \\ 20 \\ 10 \\ 5 \end{pmatrix} + \begin{pmatrix} 109 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Figure 11. Example of Matrix Model (based on Buongiorno and Michie 1980)

Buongiorno and Michie (1980) constructed a model for a north American hardwood forest, using the stationary and Usher assumptions, but not the Markov assumption. Their matrix was essentially a bi-diagonal matrix with probabilities of movement, and a vector of negative values for predicting regeneration (Figure 11). Their model also requires that the ''regeneration reserve'', an estimated parameter, be provided. The regeneration vector and regeneration reserve were estimated using ordinary least squares linear regression. Their model took the form

$$\underline{Y}_{i+1} = (\mathbf{A} + \mathbf{N}) \bullet (\underline{Y}_i - \underline{H}_i) + \underline{C}$$

where $\underline{Y}_i$ is the stand table at time i, $\underline{H}_i$ indicates the number of trees harvested from each class at time i, **A**

is a standard bi-diagonal Usher matrix, and **N** and $\underline{C}$ are derived using linear regression (equation 6.1), which was fitted using linear regression. **N** is a zero-filled matrix with the first row comprising $d_i, i = 1, \ldots, n$ estimated from equation 6.1 ($d_i = 0.27 - 9.65B_i$ where B_i is the basal area of a tree at the midpoint of class i). $\underline{C}$ is a zero filled vector with 109 (from equation 6.1) as the first element. Equation 6.1 was fitted to recruitment data using ordinary least squares linear regression:

$$R = 109 + 0.27N - 9.65B \tag{6.1}$$

where R is the total amount of recruitment (*stems*/*ha*/*ann*), n is the total stocking (*stems*/*ha*) and B is the stand basal area (m^2/*ha*).

Buongiorno and Michie's (1980) model differed from an Usher matrix in that the harvest was represented by a separate matrix rather than by including the harvest in the coefficients of the transition matrix, thus allowing the harvest to be varied. Part of the mortality was lost, even if the stand was harvested, and ingrowth was affected by stocking of the stand (basal area as well as number of stems). Their model suggested that the undisturbed development of the forest would lead to an all-aged stand with equal numbers in each size class (de Liocourt's q=1.0), and that the number of stems and basal area per hectare would follow a dampened oscillation (cycle about 200 years), gradually tending towards equilibrium.

Buongiorno and Michie (1980) found that fixed proportion harvesting would lead to the removal of about 15 square metres of basal area every 35 years. Optimum economic harvesting regimes would lead to the removal of all trees in a specific diameter class at fixed intervals. However, Mendoza and Setyarso (1986) found that selection logging which removed a proportion of trees in each merchantable size class would sustain higher yields than simple diameter limit cutting. Their model also revealed that harvesting practices in Indonesia could not be sustained, as residual stockings were too low to enable the next anticipated harvest in 35 years. These conclusions were drawn despite a number of simplifying assumptions, including taking no account of stand basal area, and simply recruiting a constant amount of ingrowth into the smallest size class.

One problem with matrix models is in accommodating regeneration. Buongiorno and Michie (1980) predicted ingrowth from the number and basal area of trees in the stand, and expressed these as negative coefficients in their matrix. Bosch (1971) predicted regeneration only on the death of another tree. This is a realistic assumption for undisturbed climax TMF, but may be inappropriate for managed forests subjected to timber harvesting.

Two major weaknesses of the matrix approaches are the stationary and Markov assumptions (see 6.2.1). These may be overcome by estimating a new matrix for each projection interval (to account for new basal area, etc.) either directly from a subset of the database or from indirectly using equations (e.g. Solomon *et al.* 1986, 1987), but that eliminates many attractions of the approach. Solomon *et al.* (1986, 1987) and Mengel and Roise (1990) avoided the Markov assumption by using a 2-stage approach in which matrix coefficients were re-computed at the start of each 5-year projection period. Coefficients were predicted as a linear function of stand basal area, diameter class and proportion of hardwoods in the mixed conifer/hardwood stands. Their model divided the stand into 13 species groups, with separate state vectors and transition matrices for each group. Twenty-five one-inch (2.5 cm) classes were recognized. Their approach was similar to that of Buongiorno and Michie (1980), and used a tri-diagonal matrix to predict the probability of moving none, one or two classes.

6.2.4 Estimation of Matrices

Michie and Buongiorno (1984) discussed four approaches for computing the coefficients for a general matrix. Their conclusion was that the simplest and most obvious method was the best. Data describing the initial stand condition are compiled, the fate of trees determined, and the actual movement of trees from the initial class is used to compile the probabilities for the matrix.

This approach fails to utilize the fact that the growth pattern of trees is such that the movement probabilities for adjacent cells of the matrix should be similar. Thus the probability from moving from class $i - 1$ to class i should be similar to that for moving from class i to class $i + 1$. If not, it is likely that insufficient data were used. This fact can be exploited to estimate entries with greater precision and/or fewer data, by using probabilistic regression (Lowell and Mitchell 1987, 1988, Bonnor and Magnussen 1988, Vanclay 1991d).

6.3 Equations

Equations offer an efficient means to overcome the Markov and stationary assumptions and ensure that predicted increments respond to stand basal area and other stand variables. The equations can be applied directly, or can be used to re-estimate coefficients in a matrix before each projection cycle (e.g. Solomon *et al.* 1986, 1987, Mengel and Roise 1990).

6.3.1 Empirical Equations

One of the simplest applications of such equations is that of Grimes and Pegg (1979) who used only three classes (20-30, 30-40, 40+ cm d.b.h.). They recorded basal areas and numbers in each class, and predicted the change due to growth, recruitment and mortality in these classes by simple empirical equations. Despite the crude nature of the model, it produced acceptable results in short term simulations.

Leak and Graber (1976) represented an uneven-aged beech-birch-maple stand with eleven diameter classes of different widths. They computed diameter increment as an empirical function of diameter and basal area, and used estimates of diameter increment to predict movement ratios for the new stand table. Empirical functions were also used to predict recruitment into the smallest class, and to predict mortality in the smaller classes. Mortality in the larger classes was assumed constant at about one percent.

Ek (1974) used 5 cm diameter classes, and computed upgrowth directly from an exponential function in relative dominance. Recruitment was expressed as an exponential function of stocking and stand basal area, and mortality reflected the relative dominance of trees. One problem with this model is common to traditional stand table projection, in that fractions of trees can grow to very large diameters.

6.3.2 Systems of Equations

Leary (1970) used three diameter classes (10-20, 20-30, 30-40 cm), expressed the relevant equations as a system and solved them simultaneously using boundary values rather than ordinary least squares. He assumed that growth in the largest class was dependent only upon the sum of diameters in that class, but that the growth of the smallest class was dependent upon the sum of diameters of all classes. Leary showed that simultaneous estimation leads to more precise estimates of the parameters, and that the parameters are more meaningful. The simultaneous system immediately makes it clear that the smaller sizes will respond with increased growth if the larger trees are removed, and this is not apparent in the simple case with independent estimation.

6.4 Stand Class Distribution Models

One of the problems with simple stand table projection and matrix approaches is that a small proportion of stems in any class may move *n* classes in *n* projection periods. This may not be a problem if the projection period is sufficiently long, but it may lead to excessive growth estimates. There are two possible solutions, namely to use more and narrower classes, or to use some smoothing of the stand table. Hann (1980) used 1 inch (2.5 cm) classes and predicted movement using empirical equations, but one problem with such small classes was that recruitment occurred in the second as well as the first class, as trees could move more than one class in one projection period. This section discusses an alternative, the smoothing techniques.

Campbell (1981) developed a model for pine plantations in Western Australia, using sixteen 8 cm diameter classes. For computational efficiency, Campbell adopted the ‘‘Usher assumption’’ that the size class and the time interval (here one year) must be such that no tree can advance more than one class in a single growth period. He fitted a series of quadratic splines to the stand diameter distribution, one quadratic equation to each diameter class, with the explicit constraints that the resulting curve must be smooth, continuous and encompass the appropriate number of trees in each class, must be positive through its range and must reach zero at the lower limit of the smallest class and the upper limit of the largest class. The

integral of these splines is easily determined, and allows precise estimates of mean diameters and total basal area within each class. It also allows the computation of numbers of trees, basal area or volume between any specified diameter limits. Unlike the Weibull distributions often used for whole stand models, the spline curve approach (Smith 1979) can accommodate stands which are not uni-modal.

Campbell (1981) used a Bertalanffy function to predict increment of the mean tree in each class, and assumed a near-normal distribution (approximated by the Weibull distribution) of increment within each class. Upgrowth from each class was estimated from the within-class distribution of trees and the predicted distribution of increments. Thus the heteroscedastic nature of growth was preserved in the model. He controlled the rate of growth in the largest class by specifying a critical fraction of the total stocking which represents the minimum amount that must be projected before a new class was initiated. If the projected upgrowth was less than this critical proportion, the upgrowth remained in the existing largest class. The same critical proportion was used to absorb the smallest class into the next class, when upgrowth had reduced the proportion remaining in the smallest class to a sub-critical amount. Campbell assumed that in a well managed plantation, mortality and recruitment would be negligible. The resulting model has been extensively used for production yield scheduling of Western Australia's pine plantations.

Because the series of quadratic splines is sufficiently flexible to accommodate a wide range of diameter distributions including multi-modal distributions, Campbell's (1981) approach can be adapted to uneven-aged stands. Vanclay (1981) adapted the approach to model uneven-aged *Callitris* forests in Queensland, using twenty 5 cm d.b.h. classes. Although the model provided useful results, the tree list approach provides greater flexibility and has been adopted for these forests (Vanclay 1988a).

Korsgaard (1989a,b) assumed that a J-curve defined by de Liocourt's quotient could describe the distribution of stem sizes in TMF. He reported that for dipterocarp forests in Asia, quotients ranged from 1.3 to 1.6 and remained relatively constant. Assuming such a distribution rather than a uniform distribution within diameter classes, results in smaller, more realistic estimates of growth. For instance, for a given diameter increment, a uniform distribution within classes may imply that half the stems in a class would move up to the next class, whilst a J-distribution with quotient 1.5 would imply that only 0.4 of the stems move (Korsgaard 1989a,b). Chai and Sia (1989) described the application of this model to mixed swamp forests in Sarawak. The method is reasonably efficient provided that few species classes are simulated. If more than a few (e.g. 4 or 5) classes are required, the method becomes inefficient.

6.5 Representative Trees Approaches

Representative trees approaches entail the modelling, explicitly or implicitly, of several trees each of which are deemed to represent a class or cohort of trees. The growth predicted for the representative tree can be used to update cohort attributes using one of three methods (analogous to stand table projection): All the trees in the cohort may be assumed to be identical to the representative tree, or they may be assumed to be uniformly distributed within the cohort whilst maintaining the same growth as the representative tree. Alternatively, some account may be taken of the distribution of stems and increments within any class (c.f. Wahlenberg's method).

The utility of the first method depends largely on the characteristics of the stand and on the number of cohorts. The second method is more-or-less equivalent to the equation approaches described previously. In the third method the distribution of stems within any cohort may be resolved by reference to the whole stand distribution or resolved within the class itself.

The representative trees approach is rather logical, and eliminates many of the problems inherent in alternative modelling strategies. Three main components of growth are modelled: diameter increment is modelled by incrementing the size of the representative trees; mortality by reducing the expansion factor (the number of trees represented by each simulated tree); and recruitment by adding a few new trees to the simulation.

6.5.1 Even-aged Monospecific Stands

Gibson *et al.* (1969, 1970) developed a model for *Pinus radiata* plantations which closely parallels the first

of the methods described above. The model commences with a count of trees in each one inch (2.5 cm) diameter class from three inches (8 cm) to 30 inches (76 cm). The model increments the class mean diameter through the use of a stand basal area increment function and an allocation rule. The numbers of trees in each class remain the same throughout the projection period, unless removed by harvesting (i.e. assume no mortality). Thus after some projection, the numbers in each class remain the same, but the classes will correspond to different sizes. These sizes need not be integer sizes, and consecutive classes need not differ by the original amount (i.e. 1 inch). Within each class, each tree is assumed to be identical to the mean tree.

Clutter and Allison's (1974) model for *P. radiata* plantations represents the third approach discussed above. They divided the stand into 25 cohorts, each initially with an equal number of trees. The median diameter for each cohort was derived by fitting a Weibull function to the diameter distribution of the whole stand, and computing the diameters corresponding to the second, sixth....., 98th percentiles. The increment and mortality functions used in the model concerned these hypothetical median trees. Trees remained in their cohort until harvesting, thus after thinning a number of the cohorts became empty, and the median trees no longer corresponded to the percentiles of the stand. All the trees in any cohort were assumed to be identical to the median tree. Alternatively, a Weibull distribution could be fitted to these 25 median trees at the end of the projection, and estimates of number, diameter, basal area or volume could be obtained for any portion of the stand.

Alder's (1979) model for coniferous plantations in Africa was a hybrid between that of Gibson *et al.* (1969) and Clutter and Allison (1974). Alder considered deciles, and modelled the development of the ten median trees corresponding to the fifth, 15th, ..., 95th percentiles of the cumulative distribution. This initial distribution could be estimated for a stand with dominant height of seven metres, using the Weibull distribution. It was assumed that there was no mortality, so Alder kept no record of trees in each cohort: it remained constant at ten percent of the total stand stocking. Diameter increment was predicted from estimates of height increment derived from the height-age relationship. Alder also assumed that all trees in any cohort were identical to the median tree; he made no attempt to reconstruct a distribution of diameters. Alder and Schneider (1979) used a similar approach, but predicted the initial stand condition at age fifteen, and assumed that two parameters of the Weibull distribution, the location and shape parameters were zero and two respectively. This results in a distribution which passes through the origin and is slightly left-skewed, and allows certain computational efficiencies. Their model allowed mortality due to suppression and windthrow.

The three methods discussed here have all been used operationally in making predictions for commercial plantation management. In general, the representative trees approach may be efficient for even-aged stands, but are inadequate for uneven-aged stands, especially where the representative trees are selected as percentiles from the cumulative diameter distribution. The diameter distribution of an uneven-aged stand can be represented with straight line segments, or with polynomial splines. However, care is needed in selecting the representative trees for efficient estimation. In uneven-aged stands, the greatest number of trees have small diameters, but the small number of large trees have greater variability in their growth (increment and mortality), and represent the greatest proportion of the value in the stand. Any attempt to select representative trees based on equal numbers in each cohort must be viewed as inefficient in such stands; better approaches are available.

6.5.2 Mixed Stands

Leary (1979) devised an approach akin to that of Gibson *et al.* (1969) which is suited to mixed stands. The emphasis of this model is on faithfully following the ''essence'' of the process of stand growth and development, and on computational efficiency. A design feature of Leary's model is that each tree is assigned to a cohort, and remains in that cohort. Leary adopted a ''relativistic'' approach to tree list partitioning, rather than the traditional metric approach. Thus cohorts contain approximately the same number of trees. If the number of trees is not exactly divisible by the number of cohorts, the cohorts are arranged so that the smaller cohorts contain more stems. Leary (1979) gave an example where the traditional metric approach used by Gibson *et al.* would result in four classes (5-9, 9-13, 13-17, 17-21 inches diameter) which would contain 12, 3, 0 and 4 trees respectively. His relativistic approach leads to three cohorts (6-8, 8-14, 14-19 inches diameter) with 10, 5 and 4 trees respectively. Leary argued that this was more efficient, leaving no empty classes.

Leary's model allowed varying levels of resolution. At the lowest level of resolution, it operates on a single cohort for each species. At the intermediate level, it identifies size, and simulates three cohorts for each species. At its highest level of resolution, each cohort represents an individual tree and the model becomes a tree list model (Section 7.3). Thus the user can select a level of resolution appropriate to his requirements and budget. This is achieved by predicting sum of diameter increments for each cohort by the potential diameter increment (Hahn and Leary 1979) appropriate to the mean tree of the cohort, reducing it by a modifier function (Leary and Holdaway 1979) which accounts for stand density and competition, and multiplying this by the number of trees in the cohort. The list of individual diameters input to the model is not discarded, but retained and at the end of the simulation each tree is updated by its share (Leary *et al.* 1979b) of the accumulated increment in its cohort. In this way Leary hoped to derive reasonable estimates of the actual diameters in the final stand.

TABLE 5. Example of Tree List Approach (Vanclay 1989a)
(Using data derived from a point sample with a 10 sq m/ha BAF prism)

Specific name	Trade name	Inventory data	Group identity	Cohort list as projected											
				Year 0		Year 1		Year 2		Year 3		Year 10		Year 25	
			(VLG)	N/ha	DBH	N/ha	DBH	N/ha	DBH	N/ha	DBH	N/ha	DBH	N/ha	DBH
Cardwellia sublimis	Northern silky oak	NSO 41 useful	322	7.39	41.50	1.85	41.97	1.85	42.30	1.84	42.64	1.83	44.99	1.81	49.96
						5.54	41.83	1.38	42.26	1.38	42.59	1.38	44.94	1.36	49.92
								4.15	42.13	4.15	42.46	4.13	44.81	4.08	49.78
Sloanea australis	Blush alder	BLA 49 useful	374	5.20	49.50	1.29	49.92	1.29	50.22	1.28	50.51	1.23	52.57	1.14	56.81
						3.88	49.79	3.86	50.09	3.84	50.38	3.70	52.44	3.41	56.68
Cardwellia sublimis	Northern silky oak	NSO 26 useless	492	18.13	26.50	18.10	26.80	18.07	27.08	18.05	27.36	17.86	29.35	17.51	33.67
Canarium baileyanum	Brown cudgerie	BRC 68 useless	495	2.71	68.50	2.69	68.85	2.67	69.18	2.65	69.51	2.51	71.82	2.24	76.60
Xanthophyllum octandrum	Macintyre's boxwood	MCB 42	495	7.05	42.50	6.99	42.82	6.94	43.13	6.89	43.43	6.52	45.56	5.81	50.11
		MCB 36	495	9.56	36.50	9.48	36.81	9.41	37.09	9.34	37.38	8.84	39.41	7.86	43.76
	Misc.	MIS 24 MIS 16	495	21.21	24.50	21.04	24.76	20.87	24.99	20.71	25.24	19.58	26.95	17.39	30.69
	Misc.	MIS 16 MIS 16	495	140.30	16.50	139.14	16.71	138.00	16.90	136.86	17.10	129.19	18.50	114.34	21.62

Reed and Clark (1979, Reed 1980) based their stochastic model for investigating succession on the approach of Botkin *et al.* (1972), but modelled the mean tree of each cohort rather than the individual trees. Their cohorts comprised trees of the same species with similar age, height, diameter, leaf biomass, etc. Trees normally remain in their initial cohort throughout simulation, but browsing damage causes initiation of a new cohort for the damaged stems. The model concerned the ecological succession within the forest, rather than timber production, and many of their functions are subjective and speculative. Browsing and mortality were the only stochastic functions in the model.

Vanclay (1989a) described a tree list model for TMF in Queensland (Table 5). These forests comprise several hundred tree species, of which more than one hundred are of commercial importance. For efficient simulation, species were grouped according to growth habit, size at maturity and tree-marking guidelines (Preston and Vanclay 1988). These groups were coded so that the appropriate growth group (5 groups) and harvesting group (9 groups) could be identified. Each cohort was characterized by its species group code, its diameter and expansion factor (stems/ha represented by that cohort). The model admitted a maximum of 200 cohorts, and the actual number of cohorts was maintained near this limit by allowing record doubling (e.g. Table 5, first record, years 1 & 2) and merging (e.g. Table 5, last record, year 0). Cohorts of small trees may contain many trees; as they attained a size of some economic importance they could split into new cohorts (one with slower-growing and one with faster-growing trees) reflecting the actual increment distributions observed in the permanent plot data. Cohorts of overmature trees were merged as their expansion factors reduced through mortality and logging. The model was subsequently enhanced so that each species retained its individual identity.

6.6 Conclusion

Stand class models have been extensively used for modelling growth and yield in uneven-aged forests, and offer several advantages. These methods are relatively simple, computationally efficient, and provide information in sufficient detail for many forest management applications. Classical stand table projection continues to be useful where summarized stand data are available and computer resources are limited. However, the many species and wide range of stem sizes encountered in TMF may require many classes, which detracts from the method. Matrix methods are easily implemented on small computers and programmable calculators, and produce good results where stand density and silvicultural practices are maintained within a narrow range, but suffer from assumptions which become untenable for long projections and for wider ranges of stand conditions. Tree list models offer the greatest flexibility, enabling projections under a wide range of conditions and enabling diverse information on predictions to be reported.

7. Individual Tree and Tree List Models

As the class width approaches the resolution of diameter measurement (e.g. 1 cm) and the number of trees per cohort approaches unity, the distinction between stand class approaches and individual tree models becomes blurred, especially for tree list or cohort models. The distinction I make between individual tree models and tree list models is largely based on the modelling of mortality. Individual tree models are defined as those models which simulate each individual tree for a given plot (i.e. expansion factor always 1.0), whereas tree list approaches may simulate a variable number of trees in any cohort (i.e. expansion factor can be any real number). This distinction has significant implications for the processing of inventory data obtained from point samples (sampling with probability proportional to size), and for the modelling of mortality. Thus an individual tree model must employ stochastic mortality prediction to maintain expansion factors of exactly one per plot, whereas a tree list approach may deterministically reduce expansion factors so that they represent less than one tree per plot.

This chapter follows Munro's (1974) distinction between distance-independent individual tree models which do not use the spatial distribution of stems, and distance-dependent models which require the spatial coordinates of each tree being modelled.

7.1 Distance-Dependent Individual Tree Models

Distance-dependent models are of limited utility for growth and yield studies in TMF since data of the necessary detail are rarely available. However, development of these models has provided a basis for other modelling approaches, and a brief review of some such models is given.

Much of the research into distance-dependent individual tree models has concerned the development and application of competition indices. Numerous such indices have been proposed, but most fall into three broad categories: the competitive influence zone, area potentially available and size-distance approaches. The competitive influence zone (e.g. Newnham 1966, Gerrard 1969) is based on the reduction in crown width compared with open-grown trees, and most indices compute a weighted estimate of the potential area overlapped by other trees. Area potentially available (e.g. Moore *et al.* 1973, Adlard 1974) is derived by tessellating the plot area into polygons representing the area potentially available to each tree; the polygons may be weighted according to tree size. Size-distance approaches (e.g. Lemmon and Schumacher 1962a,b, Spurr 1962a, Steneker and Jarvis 1963) compute an index of competition based on the angle subtended by each potential competitor at the subject tree. Empirical studies (e.g. Opie 1968, Johnson 1973, Daniels 1976, Alemdag 1978, Martin and Ek 1984, Barclay and Layton 1990) suggest that competition indices rarely provide better estimates of increment than simple measures of stand basal area, and that the expense of determining individual tree positions in the stand is rarely warranted for the purposes of yield forecasts.

7.1.1 Deterministic Models

Newnham's (1964, Newnham and Smith 1964) model for even-aged stands of Douglas-fir contained three important assumptions which have provided the basis for many similar models:

- that a tree free of competition has the diameter growth rate of an open grown tree of equal diameter,
- that a tree subject to competition has its increment reduced by an amount proportional to the level of competition, and
- that mortality occurs when diameter growth falls below a threshold level.

For computational ease, Newnham assumed that competition extended only as far as eight times the initial spacing, and this led to bias in estimates of yield from closely spaced stands. Monserud and Ek (1974) and Martin *et al.* (1977) have reviewed techniques to reduce this ''plot edge'' bias. Newnham's model gave

good results, and numerous models have been based on this approach (e.g. Bella 1970, Hatch 1971, Lee 1967, Lin 1970, 1974). However, Larocque and Marshall (1988) argued that whilst the overall performance of Newnham's model has been demonstrated to be satisfactory, the three basic assumptions have never been tested independently, and remain unconfirmed. The third assumption regarding mortality remains controversial (see Chapter 9).

Larocque and Marshall (1988) suggested that four major factors limited the ability of single tree distance-dependent growth models to predict stand and tree growth:

- failure of competition indices to incorporate sufficient biological detail to remain applicable over a wide range of stand conditions,
- lack of good data representing a range of growing conditions,
- large components of unexplained variation, and
- reliance on a limited number of mathematical approaches.

7.1.2 Stochastic Models

Whilst individual tree models allow explicit and testable hypotheses of many aspects of tree growth (e.g. Newnham 1964, Mitchell 1967, 1980), they also pose several problems in accommodating unexplainable variation in growth. Ignoring this variability by using general trends may produce untenable results (e.g. plantation at clearfall age comprising trees of identical size). One way to overcome these problems is to model some or all components stochastically.

Mitchell's (1967) model was probably the first distance-dependent individual tree model to include a stochastic element. He assumed that, for even-aged stands of white spruce in Canada:

- annual longitudinal growth of the main axis of all branches was related to concurrent height growth, regardless of suppression, and thus that the radial growth of tree crowns could be predicted from height growth subject to space limitations imposed by competing trees,
- the height of any tree could be predicted from the height of dominant trees and the relative crown width compared with open-grown trees,
- that suppression and mortality could be also predicted from relative crown width, and
- that tree diameter (d.b.h.) and bole volume could be predicted from tree height and crown width.

Thus his model is driven by height growth. Stochastic variation was introduced by varying the prediction of branch length:

$$L = \beta H^{0.75}$$

where L is the length (*metres*) of the i^{th} branch, H is the distance (*metres*) from the base of the i^{th} branch to the top of the tree, and β is distributed as a normal distribution with mean 0.548 and standard deviation 0.093. This variation influences crown width, and thus propagates through subsequent height growth predictions and all other components of the model. Although Mitchell's approach appears to work well for both even-aged stands of white spruce (Mitchell 1969) and Douglas-fir (Mitchell 1975), it has limited utility for modelling the yield of TMF since it is demanding of computer resources, height growth is not a feasible driving variable for TMF, and stem maps are infeasible in operational inventory.

The design features of Mitchell's (1967) model have provided a basis for several other models. The PLATIPUS (Plantation Timber Production and Utilization System, Vanclay 1988b) simulation model for coniferous plantations was designed to (1) simulate the growth of a stand under a wide range of environmental and silvicultural conditions, enabling analysis of forest management options; (2) simulate the effects of most silvicultural options (site preparation, spacing fertilizing, weed control, pruning, thinning and fire) on growth and on wood characteristics; and (3) simulate the conversion of trees and logs into various products (veneer, sawnwood, chips, pulp, etc.) and determine their intrinsic value. PLATIPUS incorporated many of the ideas of Mitchell (1969, 1975, 1980) in modelling branch and crown dynamics, but accommodated greater detail so that knot characteristics could be inferred. Height increment was derived from the standard site index (height-age) curves. Potential stem volume increment was predicted from crown area and length, a light index (relative area of sky seen by subject tree), and stand basal area.

Figure 12. Predicted Branch, Knot and Ring Patterns from the PLATIPUS Prototype

This potential increment was modified to account for site and silviculture, and partitioning rules used to determine the diameter increment at various points along the stem. Branch elongation was predicted from the stem diameter increment at the base of the branch, the number of branches in the whorl and the relative position of the whorl in the tree, but ceased when it reached another branch. Branch diameter at base was predicted from branch length. Most functions in the model were stochastic, and the variance-covariance matrix was used to ensure appropriate correlation between stochastic components. Preliminary trials with a prototype version containing several subjectively-determined relationships indicated reasonable results (Nielsen 1989). The prototype (Bragg 1988) produced standard tabular reports as well as graphical output illustrating tree profiles and branch, knot and ring patterns for any tree in the stand (Figure 12).

Ek and Monserud's (1974a,b) model was probably the first stochastic distance-dependent model for mixed forests. Like Mitchell's, their model used height rather than diameter, as the key variable. Potential height increment of any tree was assumed equal to that of a dominant tree, and potential diameter increment was the corresponding diameter increment of an open grown tree of the same height. These potential increments were reduced according to competition and relative crown depth. Stochastic variation introduced into both functions was assumed to be normally distributed. No attempt was made to incorporate heteroscedasticity or autocorrelation into this variation. Mortality was predicted following Newnham's use of a threshold value for diameter increment. The threshold is a function of tree size and assumes normally distributed stochastic variation. Ek and Monserud (1979) subsequently refined this to predict mortality as an exponential function of diameter, increment and competition. Regeneration was modelled from seed in a sub-model and recruited to the main model when it reached breast height.

7.2 Distance-Independent Individual Tree Models

Distance-dependent growth models offer potential for detailed investigations of silvicultural alternatives in intensively managed plantations, including aspects not possible in other modelling approaches. However, there are several problems: (1) permanent sample plot data rarely contain the detailed measurements necessary for formulating such models; (2) the cost of obtaining such detailed data restricts the application of such models to research applications rather than forest yield studies; and (3) competition indices which provide the basis for most such models rarely perform better than stand level measures of competition such

as stand basal area. In short, the expense of such a detailed methodology may not be warranted, and distance-independent methods offer an efficient alternative for yield forecasting studies.

7.2.1 Deterministic Models

Opie's (1972) model for even-aged *Eucalyptus regnans* comprised two parts. The first fifteen years were modelled using a whole stand distribution approach. When the simulated stand reached age 15, individual tree diameters were predicted from the binomial distribution, and subsequent modelling followed the individual tree approach. The annual cycle of diameter increment (allowing for heteroscedasticity and autocorrelation), tree death, and optional thinning or printout was implemented through seven key functions. These include a (mean dominant) height versus age and site index function, a stocking versus height function (used only until age 15), a basal area increment function, an increment allocation rule, a height-diameter function, a volume function, and a stocking guide (Reineke's line). Opie (1972) reported that the model gave promising results. Campbell *et al.* (1979) reported several enhancements to Opie's model.

7.2.2 Stochastic Models

The model of Botkin *et al.* (1972) was one of the first stochastic distance-independent individual tree models. It was not concerned with accurate estimates of timber yield, but with evaluating the modelling concept and providing reasonable predictions of species succession. Botkin *et al.* modelled the behaviour of the forest on single ten metre square plots, on an annual basis, using three routines to predict growth, death and birth. The growth function was entirely deterministic (and rather subjective), but the birth and death routines contained stochastic features. The death routine contained two functions, one for suppression induced death, and the other for random deaths. Both functions predicted the probability of an individual tree's death, and a pseudo-random number generator determined whether the tree actually died in that year. The birth routine also used a pseudo-random number generator to decide both the number and species of trees recruited each year. In determining the most likely outcome from model output, Botkin *et al.* used the mean of 100 simulation experiments. Botkin's approach has been adapted to model many other forest ecosystems (e.g. West *et al.* 1981, Shugart 1984). Two applications are relevant to the TMF, the Kiambram model (Shugart *et al.* 1980) for sub-tropical rainforest in Australia, and Doyle's (1981) model for TMF in central America.

Alder *et al.* (1977) and Synnott (1980) proposed a stochastic distance-independent model which could be used to model any forest for which PSP measures on at least two occasions exist. The approach assumed that all functions used in the model could be transformed and expressed as simple linear equations with normal error distributions. It was hoped that the cycle of model fitting, testing and application could be automated, so that the naive user could obtain forecasts based on any collection of PSP data. This model has not yet come to fruition.

7.3 Tree List Models

Although tree list approaches (e.g. Leary 1979, Reed 1980, Stage 1973) may be considered as stand class approaches, in many respects the tree list approach when employed with suitable resolution, may be considered an enhancement to the distance-independent individual tree approach. In effect, the individual tree model maintains a list of attributes (species, d.b.h., etc.) for each individual tree. The tree list approach does all this, but also simulates the number of trees per hectare represented by each tree record. This simplifies the deterministic prediction of mortality, as fractions (i.e. expansion factor < 1.0/plot) can be accommodated.

Growth models can also be formulated so that they can operate at any of several levels of resolution, and can thus provide whole stand, stand class or individual tree predictions according to the user's requirements. Leary (1979) described a system which could be used at various levels of resolution. In the simplest case, one equation could be used for whole stand growth estimates for stands of single species

composition. Mixed stands were modelled using two or more equations of the same form. The system was designed so that it could also function as a tree list model or as an individual tree model.

Brand (1981a) implemented Leary's (1979) tree list model in an information system. He required the user to group the trees according to size and species, and all trees in each group were assumed to be identical. When used as a stand class model (more than one tree in each cohort), the model allows the user to specify if the mortality is to be implemented deterministicly or stochasticly. When the number of trees in a cohort falls to one (per plot) or less, mortality is always stochastic (Brand 1981a). Deterministic mortality reduces the expansion factor by the predicted probability of mortality. Stochastic mortality causes all the trees in a cohort to die, if the Monte Carlo probability is less than the computed probability of mortality.

Stage (1973) assumed that a stochastic diameter increment function would account for sufficient variation in the stand so that the remaining functions could be deterministic. Furthermore, he assumed that provided there were sufficient trees in the stand, the effect of the stochastic individual tree function would not influence the stand total, and the model could be assumed to be deterministic. Where there were fewer trees, a ''record tripling'' procedure was used to simulate heteroscedastic growth. The functions used include diameter increment, height increment, crown dimensions, bark ratio, and mortality rate, all predicted using empirical functions. This model has been progressively refined (e.g. Wykoff *et al.* 1982, Wykoff 1986) and is extensively used in the management of natural temperate forests in north America.

Daniels and Burkhart (1988) described a modelling framework for single species stands which should enable compatible estimates at four levels of resolution: distance-dependent and independent individual tree levels, the size class and whole stand levels. Careful choice of variables enabled equations to be collapsed as resolution decreased. For example, area potentially available (Moore *et al.* 1973) was used at the distance-dependent level whilst the inverse of stocking was used at the distance-independent levels. At the stand class level, class mean values of tree size are used in place of the individual tree values. Stand mean values could also be used, and size distributions estimated from the normal probability density function (Strub 1976). The integrated system thus devised relied upon a common mathematical structure for models at different levels of resolution, which should ensure compatible predictions at all levels. Unfortunately, the framework as outlined by Daniels and Burkhart (1988) was specific to even-aged single species plantations, and the approach would appear to have little application for TMF.

Vanclay (1991d) described an approach which provides compatible stochastic and deterministic yield estimates using a tree list model. The method employs probabilistic functions to predict diameter increment, mortality and recruitment. In stochastic mode, the predicted probabilities are compared with one or more random numbers (depending on the expansion factor of the cohort), and the fate of the entire cohort determined accordingly. In deterministic mode, the predicted probability determines the proportion of the cohort which is incremented one centimetre, unless the expansion factor is small in which case probabilities are accumulated and the whole cohort incremented when the accumulated probability reaches or exceeds unity.

7.4 Empirical Comparisons

Few empirical comparisons between various types of individual tree models, or with other types of models have been published. Clutter *et al.* (1983:95) drew the distinction between explicit and implicit prediction of yields. Explicit prediction systems are those which include equations to predict volume per unit area directly (i.e. some whole stand models), whilst implicit systems predict basic information on stand structure and stand volume is obtained by summing the volumes of the various components (i.e. stand class and individual tree models). Lenhart (1988) compared explicit and implicit (Weibull distribution) approaches for modelling coniferous plantations, and concluded that the explicit formulation provided the more accurate estimates. However, implicit approaches have an advantage in offering more detailed information about stand structure.

Daniels *et al.* (1979) compared the predictive ability of two whole stand models with individual tree model of Daniels and Burkhart (1975). The whole stand models used were Burkhart *et al.*'s (1972) yield equation and the whole stand distribution model of Burkhart and Strub (1974). The most accurate (minimizing mean square error) yield estimates were provided by the whole stand distribution model, followed by the yield equation, and then by the individual tree model. However, all three models provided estimates of sufficient accuracy for most plantation management uses. The relative costs of the predictions were 1:25:1400 for the

yield model, the whole stand distribution model and the individual tree model respectively. Although the individual tree approach appears sub-optimal in both respects (accuracy and cost), it still may have a place in providing more detailed information than is available from other approaches.

Ek and Monserud (1979) reported a comparison of a deterministic stand class model based on equations and 5 cm d.b.h. classes, and a stochastic distance-dependent individual tree model. These comparisons used independent data from a range of plots of different sizes, stand densities and management histories. They used a variety of tests to compare the predictions from the models with the actual growth observed. In these tests, they used the average of four predictions from the stochastic model, and they used the actual spatial data. Both models showed close agreement with reality for short term predictions (5 to 26 years). The individual tree model appeared to be slightly but consistently more reliable, but whether or not this was judged to be significant depended on the test applied. Some tests suggested that the individual tree model was not significantly different from reality, and that the stand class model differed significantly from both reality and the individual tree model. A Kolmogorov-Smirnov test suggested that both functions were not significantly different from each other or reality at the five percent level. Both models gave comparable predictions for long term (120 year) predictions.

Mowrer (1988, 1989) demonstrated that computational efficiency is but one cost of complex models, and that complex models may have greater variance propagation than whole stand models. This means that any error in the inventory of initial stand condition may be magnified by methods such as individual tree models, whereas they may be unaltered or diminished by less complex models such as whole stand models. The implication is that models should not be unnecessarily complex, but should be designed to provide specific information needs.

7.5 Conclusion

Compared to the large variety of modelling methodologies following whole stand and stand class approaches, the individual tree and tree list modelling approaches are characterized by few variations. Seemingly, as the approach becomes more mechanistic, the logical way to implement the model becomes more apparent, and fewer alternatives are pursued. Thus the emphasis has been on adapting existing models to more forests rather than making more models. Prognosis (Stage 1973) is now in its fifth version (Wykoff 1986) and twelve regional variants have been implemented (Farr and Johnson 1988). Several versions of STEMS (Leary 1979) also exist (e.g. Belcher *et al.* 1982, Brand *et al.* 1988, Goodwin 1988, Hilt and Teck 1988, Miner *et al.* 1988, Swain and Turner 1988).

The tree list approach has been demonstrated and widely used for uneven-aged mixed forests in temperate and tropical climates, and appears to offer the greatest versatility of all approaches reviewed.

8. Modelling Diameter Increment

Although approaches such as whole stand models (Chapter 5) and matrix models (Chapter 6.2) may not require explicit predictions of diameter increment, most approaches suited to simulating growth in TMF including tree list and many stand class approaches require equations to predict diameter increment. These equations are normally developed using data from remeasured permanent sample plots on which the trees have been individually identified. Where such data are not available, predicting diameter increment is more complex and less precise.

8.1 What to Model

The increase in stem size of individual trees can be estimated as the increment or the future size of the stem diameter or basal area. Although arguments have been advanced in favour of all these options, all four are mathematically related and there should be little difference between the alternatives, provided that due account is taken of the error distribution.

8.1.1 Diameter or Basal Area Increment

Individual tree growth may be predicted as basal area increment or as diameter increment. Some argue that modelling basal area increment is preferable, since basal area increment supposedly resembles more closely the volume growth achieved by the tree than does diameter increment. More often it is justified on the basis that it provides a better fit to the data. However, the coefficient of determination (r^2) which is commonly used as a criterion, may not provide a valid comparison between the models (Section 4.7).

Diameter increment and basal area increment are related mathematically ($dB/dt = d(kD^2)/dt = 2kD\ dD/dt$), and any apparent differences in the goodness-of-fit may be due to differences in the error structure and implied functional relationship, rather than the superiority on one model over the other.

Bella (1971), Johnson (1973) and West (1980) all observed that using basal area increment as the response variable leads to higher values of r^2 than does diameter increment, and Johnson and West attributed this to the fact that basal area increment accounts for the initial size of the tree. West (1980) carried out a detailed empirical analysis, comparing the performance of three Australian and three North American tree species. West (1980) and Shifley (1982, 1987) found no evidence to suggest that the precision of estimates of future diameter from both diameter and basal area increment equations was not the same.

8.1.2 Future Diameter or Diameter Increment

Models for diameter increment can be expressed as yield functions ($D_n = F(D_0, t) + e_1$) or as growth functions, expressing the diameter increment over a specified (and not necessarily one year) growth period ($dD/dt = f(D) + e_2$, where e_1 and e_2 are the errors associated with the estimates). Many yield equations can be differentiated to form a growth equation and thus to provide compatible growth and yield estimates. Both the growth and yield approaches place equal emphasis on all observations, and where the measurement interval is the same for all observations, both measures should, in theory, yield the same result. However, where the measurement interval varies, growth and yield functions may produce different results because the error structure will be different. For example, the units of errors in growth function e_2 are mm/year, whilst the errors in yield function e_1 are in millimetres. Where the measurement interval does not vary greatly, the choice is probably based more on convenience than on statistics.

The growth equation poses some problems not encountered with the yield equation for data in which the

measurement interval is long or varies greatly. If the measurement interval does not exceed a few years, initial values of various tree and stand variables (e.g. tree diameter) can be used in regression analyses. However if the measurement interval becomes long or varies greatly, the initial values may introduce bias, and values corresponding to the middle of the interval should be estimated. A sensible compromise may be to for example, regress the increment on the mean diameter $(D_n + D_0)/2$ rather than on the initial diameter (D_0). The initial values of variables should suffice for most purposes where the interval between re-measures is not more than a few years.

Most modellers choose the growth rather than yield formulation for predicting diameter increment, but remain divided between predicting diameter or basal area increment. In practice, this choice is immaterial. Growth equations should be formulated to provide reasonable predictions for a wide range of tree and stand variables. Such equations can be formulated for both diameter and basal area increments.

8.2 Competition indices, Modifier functions & Allocation rules

Competition indices are an attempt to quantify in a simple expression, the effects of neighbouring plants on the growth of an individual in a forest stand. They may be expressed as an absolute value such as stand basal area, or as a relative index reflecting the actual growth of the subject tree compared to that expected under ideal conditions. Numerous competition indices have been proposed but empirical trials generally suggest that basal area is as reliable as any of the other indices (e.g. Opie 1968, Martin and Ek 1984). Many of these competition indices require the spatial distribution of trees, and are thus unsuited to distance-independent models such as stand class and tree list approaches.

One robust way to model diameter increment is to predict potential growth and use a ‘‘modifier function’’ to estimate actual increments (e.g. Ek and Monserud 1974a,b, Leary 1979, Arney 1985, Tome 1989). Whilst this has a number of attractions, it poses several difficulties. Suitable data may not be available and correlations for the modifier may be poor (e.g. Shifley 1987). One difficulty is estimating the potential growth rate. Shifley (1987) based his equation for potential growth rate on the fastest growing five percent of trees in his data. Such approaches should select data on the average growth rate of trees from the first to the last measure, as using growth between individual consecutive pairs of measures may select for measurement errors rather than real growth. An alternative which avoids this problem is to base the potential growth equation on trees assessed as open-grown or free of competition.

An alternative approach to direct prediction of diameter increments is to predict the stand increment, and distribute this among the stems comprising the stand. Stand increment may be predicted as basal area increment (e.g. Gibson *et al.* 1969, Opie 1972, Clutter and Allison 1974) or as increment in the sum of diameters (e.g. Leary 1979, 1980). Allocation rules may vary from the simple to the complex. One widely used approach is to allocate the basal area increment according to the basal area:

$$\frac{BI_i}{\sum BI_i} = \frac{BA_i{}^w}{\sum BA_i{}^w}$$

where the weights w may vary from 0.93 (Campbell *et al.* 1979) to 1.25 (Opie 1972) for *Eucalyptus regnans*. Vanclay (1988a) found that this relationship (with $w = 1$) held for even-aged stands of *Callitris*, but that in uneven-aged stands, the smaller trees got a greater share of the increment (i.e. $w < 1$), and predicted the allocation of increment from the cumulative basal area distribution with the largest diameter, the stand mean diameter and standard deviation of diameters as explanatory variables. Leary *et al.* (1979b) predicted the share of the increment in sum of diameters with a more complex relationship:

$$Log(Y + \gamma_1) = \beta_0 + \beta_1 Log(X + \gamma_2) + \beta_2 Log(X + \gamma_2)^2 + \beta_3 Log(X + \gamma_2)^3$$

where $Y = DI_i / \sum DI_i$ and $X = D_i / \sum D_i$, the βs are constants common to species and stands, and the γs depend upon species and stand condition. All these methods may provide good results in well-stocked stands with few species, but the allocation rules become complex for stands with many species, and the method is not recommended for TMF.

8.3 Explanatory Variables

Many of the explanatory variables used in plantation growth and yield models are of little relevance to models for TMF as they cannot be determined or have no meaning. Such variables include stand variables such as age, site index, top and predominant height, and mean diameter. Competition indices are useful in growth models for intensive plantations and research applications, but cannot be used in growth and yield models for TMF as the necessary spatial data are rarely available from operational inventory.

Most diameter increment functions for uneven-aged mixed stands employ variables such as species, diameter, stand basal area and some measure of site productivity (see Chapter 3). The advantage of such variables is that the model can readily predict future values for long projections. Other variables such as crown size and position (e.g. dominant, intermediate, suppressed) may exhibit a high correlation with diameter increment, but pose difficulties in predicting how these variables themselves change over time. Assuming no change in crown size and position may be valid for predictions over a few years, but is unreasonable for long simulations. Many studies have observed a high correlation between crown characteristics and the growth during the preceding period (e.g. Wadsworth *et al.* 1989), but ignore the fact that growth in the succeeding period is not so well correlated with crown characters (i.e. many tree variables are better for description rather than prediction of growth).

The basal area of trees bigger than the subject tree (or ‘‘overtopping basal area’’) has been found useful in many studies (e.g. Wykoff 1986, Shifley 1987, Wykoff and Monserud 1988), and Meldahl *et al.* (1985) found it the most useful single variable in predicting diameter increment. The use of overtopping basal area is analogous to the use of available light by Botkin *et al.* (1972). Botkin *et al.* (1972) calculated leaf weight as a constant times diameter squared, assumed that leaf area was proportional to leaf weight, and calculated ‘‘shading leaf area’’ as the sum of leaf areas ($\propto D^2$) of all taller trees on the plot. Botkin *et al.* (1972) assumed that tree height increased monotonically with diameter, and since the sum of diameters squared gives basal area, their shading leaf area is proportional to overtopping basal area. Botkin’s available light is predicted as $AL = \phi e^{-kSLA}$ and the predicted increment is the product of potential increment and the available light and other multipliers. It is equivalent to add $Log(\phi e^{-kSLA})$ or $c - kSLA$ to the log of the potential increment, which is how overtopping basal area usually enters diameter increment equations. Several studies (e.g. Ford and Diggle 1981, Cannell *et al.* 1984, Hara 1984, 1986) have indicated that competition between plants in a monoculture is mainly for light, rather than for other environmental resources. This means that competition is ‘‘one-sided’’ in these studies; that larger plants shade smaller ones, but not vice-versa. Thus overtopping basal area should be a good predictive variable and complementary to stand basal area which indicates ‘‘two-sided’’ competition for resources other than light (i.e. moisture, nutrients). Wykoff (1990) obtained better predictions by using the interaction between tree size and overtopping basal area, $OBA/Log(D+1)$, rather than OBA. He also argued that overtopping basal area was a more appropriate explanatory variable than relative size (OBA/BA and $D/\bar{D}$) as the latter would be influenced by harvesting and would predict a counter-intuitive response to thinning from below.

Ek and Monserud (1974a,b), Daniels and Burkhart (1975), Ek and Dawson (1976), Alder (1979) and Alder and Schneider (1979) predicted diameter increment from height increment, assuming that for even-aged stands, height increment is independent of stocking. Mitchell (1969) modelled height and crown development and predicted the current diameter of any tree from height and crown width. These approaches have been useful for modelling regeneration and for monospecific even-aged coniferous forests, but one major problem in TMF is that heights are more difficult and inaccurate to measure than diameters.

8.4 Diameter Increment Functions

A vast number of diameter growth and yield functions have been published and no attempt is made to review even a small number of these. Only those functions which do not require age are considered. Functions are grouped into broad classes (empirical, theoretical, probabilistic) to allow generalizations to be made. Little distinction is made between diameter and basal area increment models, or between growth and yield models.

8.4.1 Empirical Functions

Empirical equations are simply mathematical expressions with a resemblance to the observed growth, but without any associated hypotheses of causation or explanation of the phenomenon. Such equations may be useful for interpolation, but contribute little to a greater understanding of the processes of growth, and may be unreliable when extrapolated beyond the limits of the data (e.g. Payandeh 1983). However, empirical equations can be formulated to provide biologically realistic predictions across a wide range of values (i.e. beyond the limits of the data) (e.g. Wykoff 1990), and such equations may provide better predictions than theoretical equations (e.g. Martin and Ek 1984). They may also be easier to fit to the data. Many empirical equations are developed using stepwise regression analysis to select variables correlated with the response variable. Unfortunately, such equations often include an unnecessarily large number of variables, and may perform poorly when used near the limits of the data.

One widely used empirical equation is the simple quadratic

$$DI = a + bD + cD^2$$

This equation has been used by Leak and Graber (1976), Alemdag (1978) and West (1980, 1981). Unfortunately, it may not provide robust results, as the equations of Alemdag and West contained a minimum, but no maximum and thus predicted ever-increasing increments for larger and larger trees. Quadratic equations may provide reliable estimates of growth over a limited range of diameters, but are unsuitable for use in long term growth simulation models.

Mawson (1982) recognized these limitations of the popular quadratic equation, and proposed a variation of Schumacher's (1939) equation:

$$Log(DI) = a + b/D$$

He argued that the parameter a could be expressed as a function of site, and the parameter b as a function of previous stand treatment. As this equation predicts increments asymptotic to e^a as D becomes large, care should be taken that sensible values are estimated for parameter a.

Hilt (1983) used a two-stage analysis to establish diameter increment functions for even-aged oak forests. The first stage fitted $BI = \beta D^2$ (BI is tree basal area increment and D is d.b.h.) to each plot, and the second stage fitted

$$Log\beta = \gamma_1 + \gamma_2 LogSI + \gamma_3 \bar{D} + \gamma_4 PS$$

to each plot, where SI is site index, $\bar{D}$ is the quadratic mean stand diameter and PS is percent stocking. Thus his final model was, expressed as a diameter increment function:

$$DI = \alpha SI^{\gamma_2} e^{\gamma_3 \bar{D} + \gamma_4 PS} D$$

Whilst this function performed well for tree sizes in the calibration data, it may predict excessive increments for large trees.

Zeide (1990) argued that growth equations should have ''an upper asymptote to express the fact that any growth is limited. Non-asymptotic growth is always temporary and can be rendered by a segment of an asymptotic model. In this sense, the linear, logarithmic, elementary exponential, and other non-asymptotic equations cannot be considered as growth equations''. However, one empirical equation which can be fitted easily has these properties, and relies on the underlying relationship:

$$Log(\Delta D^k) = \beta_0 + \beta_1 Log(D) + \beta_2 D^k$$

where D is tree diameter and typically $k = 1$ or $k = 2$.

8.4.2 General Theoretical Functions

General theoretical equations are based on assumptions which give at least a tentative explanation of the mechanism of growth in general and *a priori* terms (Sweda and Koide 1981). There is no particular theoretical equation which relates specifically to the growth of trees, and the Bertalanffy equation for the growth in weight of animals is rather quasi-theoretical when applied to the diameter growth of trees:

$$\frac{dD}{dt} = nD^m - pD = pD\left\{\left(\frac{D_{\max}}{D}\right)^{1-m} - 1\right\}$$

Despite this, the Bertalanffy and similar equations have been widely used to predict diameter increments, and generally yield satisfactory results. The Bertalanffy equation is the generalized case of a number of other theoretical equations (Rawat and Franz 1974, Sweda and Koide 1981) suitable for predicting tree growth and yield. The Bertalanffy equation overcomes many of the shortcomings of empirical equations, and provides for an asymptotic maximum diameter which can never be exceeded (e.g. Shifley and Brand 1984). It also has the advantage of being a compatible growth and yield equation, and as a special case of the Bernoulli equation is one of the few nonlinear functions which can be solved in closed form (Leary 1970).

Martin and Ek (1984) considered a modification of the Bertalanffy equation for *Pinus resinosa* plantations:

$$\frac{dD}{dt} = (0.2832D^{2/3} - 0.04925D)e^{-0.03922BA}$$

but found that carefully formulated empirical equations could provide more accurate predictions within the range of the data. Shifley (1987) used a similar equation to model potential growth of many species (e.g. for eastern red cedar):

$$\frac{dB}{dt} = (0.0124B^{0.515} - 0.0149B)(0.397 + 0.00236SI + 0.0749CR)$$

where B is tree basal area (m^2), SI is site index (m) at age 50 and CR is crown ratio and takes a value in the range 1 to 9. This was fitted using a two stage approach. The first stage fitted $dB/dt = \beta_1 B^{\beta_2} - \beta_3 B$, and the asymptotic maximum size ($A = (\beta_1/\beta_3)^{1/(1-\beta_2)}$) was compared with the national register of big trees. If the asymptote seemed unreasonable, the parameter β_3 was revised to take the value $\beta_3 = \beta_1 A^{\beta_2 - 1}$. The second stage estimated the final three parameters in the equation. These potential increments were reduced by a modifier predicted from tree size, overtopping basal area and stand basal area.

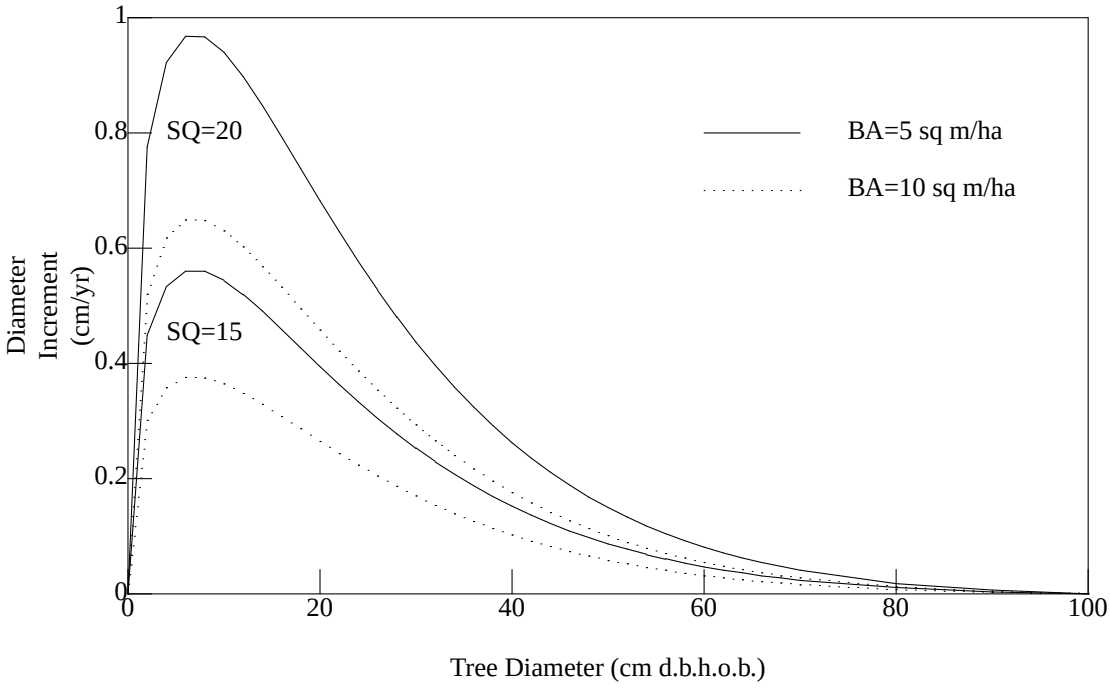

Figure 13. Potential Diameter Increment for *Callitris* (Vanclay 1988a)

Although the Bertalanffy equation is flexible and frequently used to model tree growth, it has limitations

which may be attributed to its origins as a particular theoretical equation for growth in weight of animals. When applied to diameter increment data of forest trees, the equation may over-estimate the increment of large trees. This phenomenon may be largely attributed to the fact that although an animal consists almost entirely of living tissue, a tree ''is a thin layer of living sepulchre enclosing a growing corpse'' (Harper 1977:218). Thus an asymptotic relationship may describe tree respiration better than the linear relationship assumed in the Bertalanffy equation.

Zeide (1990) argued that diameter increment asymptotically approaches zero as diameter increases to very large sizes, and suggested that the above equations may yield biased estimates for large trees. Vanclay (1988a) suggested a modification of the Bertalanffy equation which provided better predictions for large *Callitris* trees, and was asymptotic to 100 cm diameter (Figure 13):

$$\frac{dD}{dt} = (-0.06357 + 0.007809SQ)e^{-0.08006BA}D\left\{\left(\frac{100}{D}\right)^{0.5258} - 1\right\}e^{-0.04421D}$$

where D is d.b.h. (cm), BA is stand basal area (m^2/ha) and SQ is site form (m). This equation predicted increments of trees subjectively assessed to be dominant or free of competition, and thus indicated the potential rather than the average increment.

Another variation of the Bertalanffy equation is the equation of Hahn and Leary (1979) and Leary (1980):

$$\frac{dD}{dt} = a + b \times D^c + d \times SI \times CR \times D^e$$

where D is diameter, SI is site index and CR is crown ratio. This equation includes an intercept a, which improves the prediction of increments of very small trees, but detracts from the theoretical significance of the other parameters. Unlike the mass of an organism, tree biomass is not zero when d.b.h. is zero, so the Bertalanffy equation may underestimate increments for small trees. Hahn and Leary (1979) and Leary (1980) added an intercept term, but it is theoretically preferable to use an allometric relationship of $D + \gamma$ where γ is some parameter to be estimated rather than an intercept. Their equation also assumes that the site index and crown ratio affect only anabolism (photosynthesis) and do not affect catabolism (respiration); this enables the increment pattern (as well as the rate) to vary with site and crown ratio.

Botkin *et al.* (1972) devised a similar equation which can be fitted easily with minimal data:

$$\frac{d(D^2H)}{dt} = \alpha D^2\left(1 - \frac{DH}{D_{\max}H_{\max}}\right)$$

or

$$\frac{dD}{dt} = \beta D \frac{1 - \dfrac{DH}{D_{\max}H_{\max}}}{2DH + D^2\dfrac{dH}{dD}}$$

Botkin *et al.* (1972) used a simple quadratic to represent the height-diameter relationship (Ker and Smith 1955), leaving only three parameters to be estimated, β, $D_{\max}$ and $H_{\max}$. Two of these, $D_{\max}$ and $H_{\max}$ could be subjectively estimated by observation or by consulting the literature, and consequently Botkin needed only to estimate one parameter from the data. In fact, Botkin estimated this parameter from $H_{\max}$ and $D_{\max}$, by assuming that trees attained two thirds of their maximum size at half their maximum age, or by choosing the parameter such that $\beta = 5DI_{\max}H_{\max}/D_{\max}$. Thus Botkin could estimate all his coefficients from $H_{\max}$, $D_{\max}$ and either maximum age or maximum observed increment, and no coefficient needed to be determined by regression analysis. However, he was interested in modelling species succession rather than timber yields, and thus tree size and growth prediction was of lesser importance in this study than it is in forest management for timber prediction. Botkin *et al.* (1972) multiplied this potential increment by modifiers to account for shading, climate and soil quality. Vanclay (1983) demonstrated that the Bertalanffy, Leary and Botkin equations all predict a similar diameter increment pattern (Figure 14).

8.4.3 Empirical Analogues of Theoretical Functions

Several empirical functions mimic the shape of these general theoretical equations, and may offer some advantages in ease of fitting. Wykoff (1990) discussed enhancements to the diameter increment functions

Bertalanffy: $dD/dt = 0.245D^{0.44} - 0.0147D$

Botkin: $dD/dt = (D - D^2(137 + 50.9D - 0.167D^2)/611677)/(2.74 + 1.527D - 0.00668D^2)$

Hahn & Leary: $dD/dt = 0.4768 - 0.0000016D^{2.584} + 0.1452D^{0.08385}$

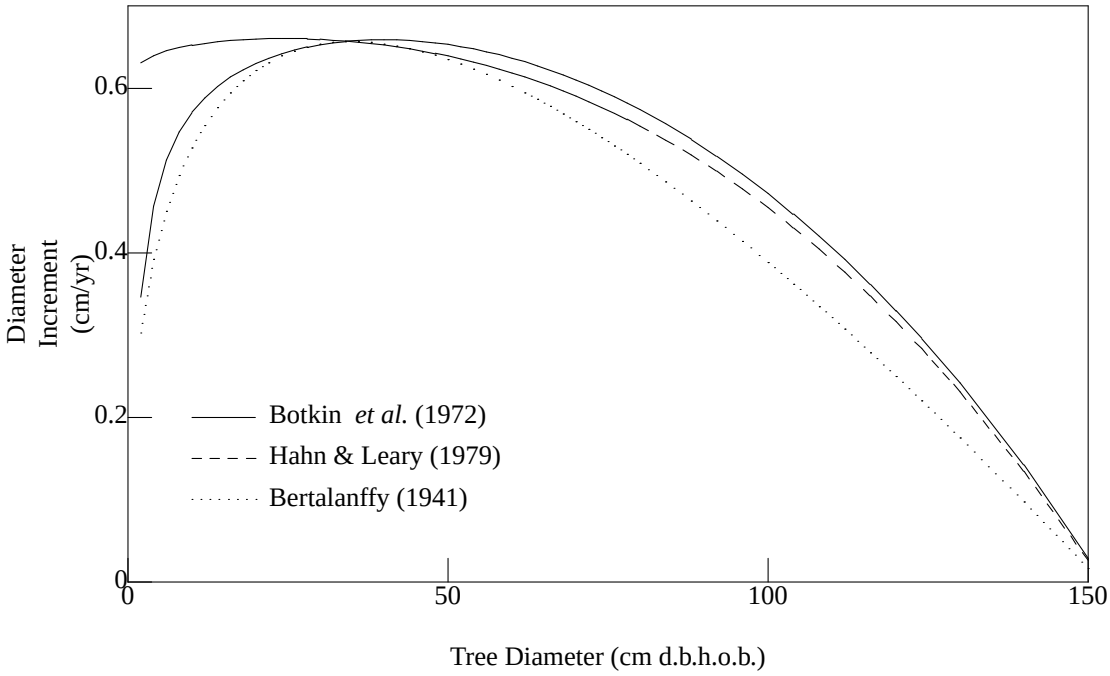

Figure 14. Comparison of Theoretical Diameter Increment Functions

in Prognosis (Stage 1973). His revised increment function took the form

$$Log(\Delta D^2) = SITE + COMP + \beta_1 Log(D) + \beta_2 D^2$$

where D is tree diameter, *SITE* describes the site effects and is estimated from habitat type, location, elevation, slope and aspect, and where *COMP* describes competition and is estimated from crown ratio, crown competition factor (Krajicek *et al.* 1961) and relative size expressed as overtopping basal area. The final function was:

$$\begin{aligned} Log(\Delta D^2) = {} & \beta_0 + \beta_1 Log(D) + \beta_2 D^2 \\ & + \beta_3 SL[cos(ASP)] + \beta_4 SL[sin(ASP)] + \beta_5 SL + \beta_6 SL^2 + \beta_7 EL + \beta_8 EL^2 \\ & + \beta_9 CR + \beta_{10} CR^2 + \beta_{11} OBA/Log(D+1) + \beta_{12} CCF \end{aligned}$$

where D is diameter, *SL* is topographic slope, *ASP* is aspect, *EL* is elevation, *CR* is the ratio of crown length to total tree height, *OBA* is overtopping basal area, and *CCF* is crown competition factor. The first line of this equation reflects the effect of tree size on increment, the second line is a proxy for site productivity, and the third line accounts for competition. This equation produces diameter increments similar to those produced by the theoretical equations, and is constrained to produce sensible increment predictions for any tree size and any stand density.

Another empirical linear equation (Vanclay 1989c, 1991b) with a similar shape does not require crown characteristics, and may provide good predictions for TMF (see Figure 6, Chapter 3):

$$Log(DI + \alpha) = \beta_0 + \beta_1 D + \beta_2 LogD + \beta_3 SQLogD + \beta_4 LogBA + \beta_5 OBA$$

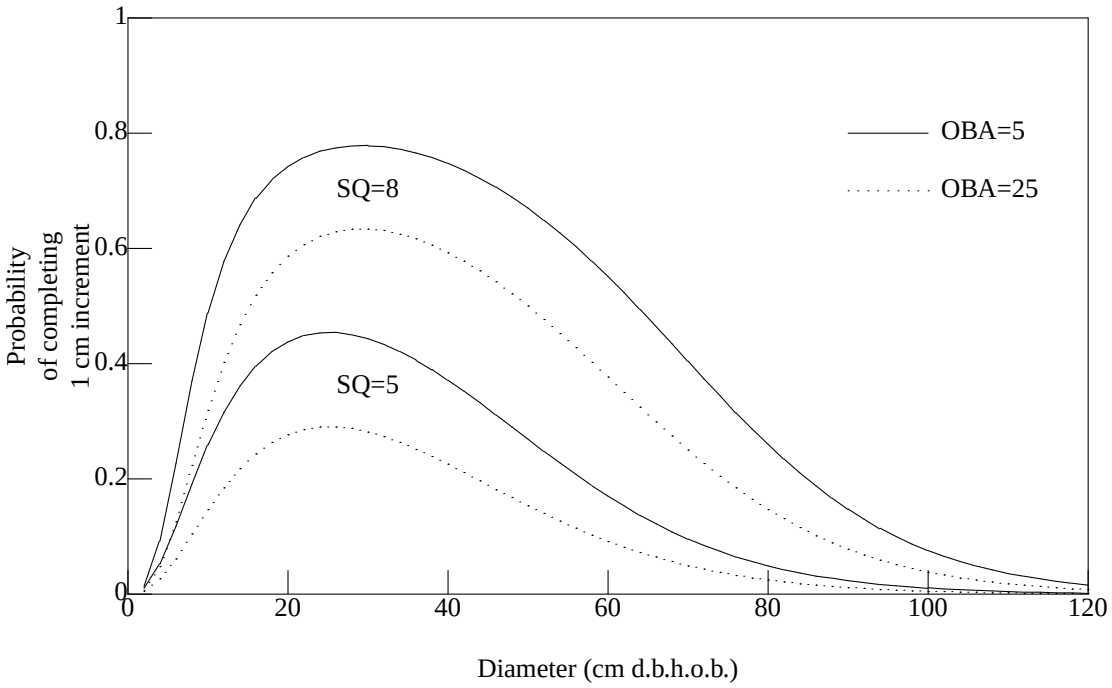

Figure 15. Diameter Increment Probabilities for *Flindersia pimenteliana*

8.4.4 Probabilistic Functions

Lowell and Mitchell (1987, 1988) used a probabilistic function to predict diameter increment, an approach which enables simultaneous estimation of increment and mortality. They claimed that as growth and mortality are biologically related they should be modelled simultaneously. Lowell and Mitchell (1987, 1988) illustrated the approach for 4 species groups in even-aged mixed oak forests in The USA. Their equation for white oaks was

$$P = \left(1 + e^{-8.901 + 271.1DI - 2.733D - 1.594LogD/\sum D}\right)^{-1}$$

where P is the probability that a tree of D cm d.b.h.o.b. will achieve a diameter increment exceeding DI cm over a five year period ($\sum D$ is the sum of diameters D of all trees per hectare, $D/\sum D$ can also be expressed as the relative diameter $D/\bar{D}$ divided by the stocking of stems/ha). The probability P_0 of achieving a zero increment is the probability of survival, $1 - P_0$ gives the predicted five year mortality. Since DI enters the equation in a linear form, the model will predict a normal distribution of diameter increments. Some transformation such as $Log(DI)$ would be more appropriate for TMF.

Vanclay (1991d) illustrated a similar probabilistic diameter increment function for TMF. He predicted the probability that a tree would complete one centimetre of growth during a given year (i.e. that a tree less than n cm d.b.h. would attain a size of n cm or more within a one year interval, for any integer n) using the equation, illustrated here for *Flindersia pimenteliana* (Figure 15):

$$P = \left(1 + e^{-0.7378 + 0.1079DBH - 1.987Log(DBH) - 0.1455SQLog(DBH) + 1.994Log(SBA) + 0.03548OBA - 0.4221PS}\right)^{-1}$$

where P is the predicted probability, PS is preferred soil, a binary variable which takes the value one on soils derived from recent alluvial, volcanic or granitic parent material, and zero on soils derived from sedimentary or metamorphic parent materials. The other variables are as previously defined. This formulation offers two advantages over alternatives: it is robust in the presence of outliers and simplifies the construction of compatible deterministic/stochastic growth models.

8.5 Diameter Increment as a Stochastic Process

Stochastic variation in diameter increment may be introduced into a model by adding random variates to predicted increments, or by using a swindle to achieve the same effect deterministically (e.g. Stage 1973).

Opie (1972) computed his increment as $BI_i = BI \times (1 \pm 0.003 \times SI)$ where BI represents the basal area increment calculated for a tree of given size, and BI_i the increment actually allocated to tree i. The sign $\pm$ in the equation was alternately positive and negative, introducing autocorrelation into the predicted increments. Site index was included to account for the greater assertion of dominance observed on better sites.

Stage (1973) assumed that the only function in his model where variation was significant, was in the prediction of diameter increment, and that this variation would have ramifications on all other aspects of the model. He assumed that distribution of diameter increments was normal with mean zero about the logarithm of the basal area increments. The logarithmic transformation ensures that the variation results in heteroscedasticity, and that negative increments cannot occur. The variance of the normal distribution was calculated from regression estimates. In stands with a large number of trees where the effect of the stochastic element was not likely to influence the total stand estimate, uniform random variables were associated with each tree, and the corresponding normal deviate added to the logarithm of its increment. Because of the logarithmic transform, the effect of this is multiplicative. The random variable associated with each tree was retained to preserve the appropriate serial correlation in increment estimates. Where the stand was represented by few cohorts, variation was introduced deterministically by ''tripling'' the number of cohorts, one each with $u - 1.549s$, $u - 0.1423s$, and $u + 1.271s$ representing the mean of the 15%, 60% and 25% population fractions (Stage 1973). Bailey and Dell (1973) observed that if the parameter c of the Weibull distribution is set as 3.6, the distribution is approximately normal and non-negative, and that the remaining parameters are easily determined. Campbell (1981) assumed that the variation in diameter increment within any class would be symmetrical about the class mean increment, and used the Weibull with these properties to predict the distribution of increments within a diameter class. Vanclay (1988a, 1989a, Vanclay and Preston 1989) adopted a similar but more simplistic approach, ''doubling'' cohorts containing many stems or exhibiting large increments.

Failure to preserve autocorrelation in individual tree models may lead to underestimates of yield, but in stand class models this may be irrelevant. Swaine *et al.* (1987) reported significant correlation between successive increment observations in TMF and found that these persisted for several years. Hilt (1983) predicted the standard deviation for a given increment prediction as $\sigma = \beta_0 e^{\beta_1 D}$ and drew random numbers to assign increments to individual (simulated) trees. He observed a correlation of 0.63 between successive increments and assumed the bivariate normal distribution in calculating successive increments.

8.6 Conclusion

No attempt has been made to review the vast number of functions used to predict diameter increment. Rather, a few equations used to predict diameter increment have been selected from the literature to illustrate strengths and weaknesses of some popular relationships. The selection of an appropriate equation may depend upon the data available and the resources available for analysis. It is irrelevant whether growth or yield, basal area or diameter is modelled. What is important is that the researcher takes proper and explicit account of the error distribution, and ensures that the function is inherently constrained to ensure reliable results over the full range of possible trees sizes and site and stand conditions.

9. Mortality and Merchantability Functions

Many growth models avoid the problem of predicting mortality by assuming that no mortality occurs in well managed plantations. This assumption is reasonable for intensively managed plantations, but is inappropriate when modelling TMF where mortality is significant and should be modelled. Stage and Renner (1988) found that most (80%) of the variability in volume predictions in temperate forests were due to uncertainty in mortality estimates. Thus modelling mortality has a major influence on the precision of forecasts.

9.1 The Nature of Mortality

The evaluation of mortality models is easier if the nature of mortality is explicitly considered. Mortality may be classified according to causal agent, or by pattern of occurrence.

9.1.1 Agents of Mortality

Age and size of trees may be factors contributing to tree death, but are rarely the actual cause of death. Competition and suppression are the agents given the greatest attention in forest modelling. Reineke (1933), Yoda *et al.* (1963) and others (e.g. Drew and Flewelling 1977, Aikman and Watkinson 1980, White 1981, Smith and Hann 1984, Westoby 1984, Lonsdale 1990) have made important contributions to the understanding of the onset of competition induced mortality in even-aged stands. In mixed forests, the situation is more complex, but the need for light, nutrients and physical space continues, and any reduction below the minimum requirements will eventually lead to death.

Pests, pathogens and diseases may include a multitude of factors which may contribute to the death of the tree. In TMF, these frequently occur in a harmonious equilibrium, with only an occasional old or suppressed tree succumbing and dying. However, epidemics occasionally occur with catastrophic effects on the forest. The effects of such an epidemic may be the comparatively mild reduction in increment for a year, or may completely eliminate the host species from the region.

Weather is another major agent of mortality in forest stands. Drought or prolonged waterlogging may hasten the demise of aged, diseased or suppressed stems, but may also cause death without such associated factors. Lightning periodically causes the death of isolated individual trees. Hail may cause damage to trees and cause loss of increment, or on rare occasions, cause death of groups of trees with extensive losses over considerable areas. Wind damage may be catastrophic, and may or may not be dependent upon stand condition. Hagglund (1981) included trees severely damaged or deformed by wind or snow in estimates of mortality. Wildfire may also cause catastrophic losses. Many other deaths recorded in PSP data can be best attributed to chance, as no satisfactory explanation can be given.

Mortality or removal of trees from a forest stand as a result of human interference is easier to deal with in growth models. The most important aspect of human interference is harvesting of trees for timber. Another important result of forest management is silvicultural treatment, where the undesirable stems are felled, girdled (ringbarked) or poisoned. From a growth modelling viewpoint, treatment can be considered in the same way as logging. Damage arising from human interference may also contribute to mortality in forest stands. Logging may destroy smaller stems, machinery may damage roots and bark and create entry points for disease and decay, or logging may simply disturb the canopy and alter the conditions to make them less favourable to a tree species, or more favourable to a pest or disease.

9.1.2 Patterns of Mortality

A growth model should account for as much of this mortality as possible. Because of the great diversity of mechanisms of death, it is convenient to group them according to the pattern of occurrence.

The term ‘‘regular mortality’’ will be used to refer to mortality that may be expected in relatively small numbers of trees (both individuals and small groups) at any time. This is something of a misnomer, as the mortality considered here is not regular, but it is a convenient and widely used term. Regular mortality refers principally to ageing, suppression and competition, but also to mortality arising from ‘‘chance’’, normal incidence of pests and disease, and weather conditions (lightning, drought and storms) which commonly occur (events with a probability of occurrence of, say not less than once in ten years). Lee (1971) used the term exclusively for deaths due to competition for limited resources, and Hagglund (1981) used the term only for individual deaths: grouped deaths are termed ‘‘irregular’’. Here ‘‘regular mortality’’ is used in a much broader context.

Management induced mortality includes those deaths and removals which occur as a result of human interference. This clearly includes logging and treatment, but also mortality arising directly and indirectly from logging damage.

Catastrophic mortality includes wildfire, the occasional but severe losses from ‘‘freak’’ weather conditions, and major pest and disease outbreaks.

9.2 Regular Mortality

Regular mortality includes most forms of mortality which may be expected to occur in a forest stand in the absence of human interference, including ageing, competition and suppression, ‘‘chance’’ mortality, low incidences of pests and diseases, and normal losses due to weather conditions. Theoretical approaches such as Reineke’s (1933) self thinning line or threshold increments would normally only predict competition-induced mortality, but could be modified to estimate regular mortality. Alternatively, other more empirical approaches based on stand or tree variables can be used.

9.2.1 Theoretical Approaches

Many growth models for even-aged forests predict mortality using the theories of Reineke (1933) and Yoda *et al.* (1963). Although such approaches provide a convenient means of estimating competition induced mortality in whole stand models, they fail to account for other components of regular mortality and do not indicate the size of trees that die. Thus they are of limited utility in stand class and individual tree models. Opie (1972) used Reineke’s line, and killed every third tree commencing from the smallest, until the required reduction in stocking was obtained. In addition, Opie assumed that any trees smaller than one-seventh of the largest d.b.h. in the stand would die. Campbell *et al.* (1979) later modified the stocking density relationship, and removed every sixth tree from the smallest, up to a maximum of 100 deaths per hectare in any year. Ferguson and Leech (1976) warned that Reineke’s line is overparameterized and that ordinary least squares methods will lead to biased estimates of parameters. Mitchell (1975) adopted Wilson’s (1951) stocking guide which relates stocking to the square of the site height (tallest 40% of trees), and assumed that the shortest trees would die. Arney’s (1985) approach was based on crown competition factor (CCF) (Krajicek *et al.* 1961) but closely follows Reineke’s line.

An alternative is to predict limiting conditions directly from growing space, competition index or crown dynamics. Mitchell (1969) modelled the crown development of trees, and assumed that when the actual crown width fell below seventeen percent of the potential open growth crown width for a tree of that size, it had a fifty percent probability of being overtopped and dying. Arney (1972) assumed that trees would die if the crown depth receded to less than five percent of the tree height. Bella (1970) predicted individual tree mortality as a linear function of height increment and competition index. Keister (1972, Keister and Tidwell 1975) predicted mortality from competition index and number of competing trees. Lee (1971, 1974) used the compound interest formula to predict reduction in stocking due to mortality in even-aged stands, and predicted the rate from stand mean diameter, which was a better descriptor of mortality than

was age. Stage (1973) used Lee's formula to predict the amount of mortality, and distributed it so that most of the deaths occurred amongst the smaller trees in the stand.

Another approach widely used for uneven-aged stands is to compare predicted increments with some threshold increment. Newnham (1964) and Strand (1972) assumed that all trees with predicted increments less than their assumed threshold would die. Reed and Clark (1979) assumed that a proportion of such trees would die. Ek and Monserud (1974a,b) used a stochastic function of diameter to predict the threshold, and any trees with a relative increment less than this died. Lin (1970, 1974) assumed that trees died after six years of continuous suppression.

The argument makes sense and is supported by some empirical evidence. Spurr (1962b) reported that any *Pinus radiata* tree achieving an annual increment less than twenty square centimetres has, on average, less than eight years to live. Bevege (1972) observed that stems dying in a *P. elliottii* plantation had existed for a considerable period (3 to 10 years, mean 6) with negligible increment, and this moribund period was independent of tree age and size. Reed and Clark (1979) reported observations that suppressed Douglas-fir trees near death achieved a radial growth of only two or three cells of xylem per year. Swaine *et al.* (1987) observed that mortality in semi-deciduous TMF in Ghana was significantly higher in trees failing to achieve measurable diameter increment —twice and four times the average rate for trees exhibiting increments in diameter of 1 and 2 millimetres per year respectively.

These observations show that the dead trees had been moribund for several years prior to death: regrettably they do not prove that failure to achieve a critical increment for several years results in death. Newnham (1964) reported that Douglas-fir trees may attain a diameter increment less than 0.3 millimetres per year for more than twenty-five years and still survive. The author is aware of *Araucaria cunninghamii* seedlings in an abandoned nursery which are over fifty years old and still have not attained a height of 0.5 metres or a root collar diameter of one centimetre. Hartshorn (1975) recorded that some dead stems in his study exhibited very little increment in the years prior to death, others had grown rapidly until shortly before death, whilst some had died while rapidly growing.

One of the problems of these theoretical approaches is that they do not account for all regular mortality in the stand. Hegyi (1974) determined the amount of mortality from a stocking guide and assumed that eighty percent of this mortality would occur in trees suffering from the greatest competition, and that the remaining twenty percent would occur randomly through the stand. However, if the stocking guide suggested that no mortality would occur, any trees whose competition index was below a specified limit were assumed to die. Opie (1972) also assumed some ''chance'' mortality, killing one tree in every 75 each year. Campbell *et al.* (1979) later revised this to a step function which predicted 1.13 percent mortality to age 15, 0.69 percent to age 40, and zero in older stands. In addition to mortality from competition, Mitchell (1969) assumed that sixteen of the smallest trees per hectare would die for every metre of stand height growth attained. Hartshorn (1975) reported that fifty percent of juvenile mortality in his study could be attributed to physical causes.

One of the problems with predicting mortality according to the casual agent (competition, etc.) is that it is frequently difficult to determine the agent responsible. The assumption that most mortality can be attributed to competition may not always be reliable. An alternative is to adopt a more empirical approach, predicting a composite estimate of all regular mortality from stand or tree characteristics.

9.2.2 Empirical Stand-level Approaches

Most stand table projection approaches employ an empirical approach, and the methods based on equations generally express mortality as a function of stand density (number of trees or basal area per hectare) (e.g. Moser 1972, Leak and Graber 1976, Grimes and Pegg 1979, Shifley *et al.* 1982). Stiell (1974) used a simple linear function of dominant height and number of stems per hectare. Hagglund (1981) predicted the proportion of the stand basal area dying in any year as a simple linear function of stand basal area.

Glover and Hool (1979) predicted mortality from relative dominance (tree basal area over stand mean), using the Weibull function. The three parameters of the Weibull function were highly correlated with stand basal area. West (1981) also predicted mortality from relative dominance, but also used a cubic function of stand age. All these functions may give anomalous results (negative mortality, or mortality greater than 100%) if used beyond the range of data for which they were developed.

Leak (1969) suggested that the proportion of mortality in an even-aged stand could be predicted from an exponential function of stand mean diameter increment during the elapsed period. In effect, he assumes that every centimetre increase in stand mean diameter is associated with a fixed decrease in stocking: in his case, fifteen percent for commercial species, and nineteen percent for intolerant and intermediate species.

9.2.3 Empirical Tree-level Approaches

Botkin *et al.* (1972) predicted the probability of mortality for any tree as a function of its potential maximum age, based on the subjective assumption that only two percent of trees reach this age. A higher rate of mortality was assumed for trees in the understorey.

Paille and Smith (1970) compiled mortality tables by tree age, relative diameter, relative height and crown class. Landford and Cunia (1977) used a linear function of increment and diameter to predict the probability of mortality, and used this as a stochastic function. Ek (1974) predicted the mortality in any diameter class as a function of the relative dominance of that class. Such functions may not provide good predictions when extrapolated, and may in extreme cases predict mortality outside the feasible range (0,1).

Hamilton (1974, 1980) and Monserud (1976) argued that it is inappropriate to estimate relative mortality using linear functions, as these are not constrained in the interval (0,1) and that logistic function is more appropriate. They suggested the logistic function, which can be expressed in several ways:

$$P = \left(1 + e^{-f(X)}\right)^{-1} = 1 - \left(1 + e^{f(X)}\right)^{-1} = \frac{e^{f(X)}}{1 + e^{f(X)}} \qquad (9.1)$$

where P is the probability of survival and $f(X)$ is some linear (or non-linear) function of several explanatory variables. Probability of mortality may be obtained from $(1 - P)$. The advantage of predicting survival is that it is, unlike mortality, a Markov process (i.e. it is a transient rather than absorbing state), and survival over an n year period can be obtained from the nth power of the annual probability of survival.

This equation may be converted to a linear equation if the data are grouped into categories with at least one death and one survival in each category:

$$Y = Log\left(\frac{P}{1-P}\right) = Log\left(\frac{N_{surv}}{N_{dead}}\right) = f(X)$$

where P is the proportion of trees surviving, N_{surv} is the number of survivors and N_{dead} is the number of deaths. This is useful for initial investigation of variables and for plotting data, but is inefficient for parameter estimation and inappropriately affects the error response surface.

Hamilton and Edwards (1976) showed how to weight equation 9.1 by the remeasure interval to account for unequal intervals. Monserud (1976) suggested that remeasure interval should be used as an exponent in equation 9.1 (i.e. use $-yrs$ rather than -1) rather than as a weight. Whilst Monserud's approach is technically correct, the alternative of Hamilton and Edwards (1976) provides an efficient approximation which is reasonably accurate provided that remeasure intervals do not exceed 7–8 years and mortality is less than about 0.7 percent (Hamilton, *pers. comm.*).

Hamilton and Edwards (1976) fitted equation 9.1 for several species using empirical functions of diameter, height, age, defect, crown class and stand basal area. Relative size of individual trees may also be useful, and has been included in mortality models as the ratio with mean diameter ($D/\bar{D}$) (e.g. Hamilton 1986, 1990) and as the relative position in the cumulative stand basal area distribution (OBA/BA) (e.g. Vanclay 1991c). Monserud (1976) found that many tree variables were highly correlated and provided equally good predictions of mortality when used separately, but offered no further improvement when more than one variable was included (e.g. tree height and diameter). Vanclay (1991c) predicted survival from tree size, site and competition (Figure 16):

$$\hat{P} = \left(1 + e^{-(\beta_0 + \beta_1 Log(DBH) + \beta_2 DBH + \beta_3 RS^3 + \beta_4 SQ + \beta_5 BA + \beta_6 Log(BA))}\right)^{-1}$$

where P is the annual probability of survival, DBH is tree diameter, RS is relative size denoted by the overtopping basal area divided by the total stand basal area (OBA/BA), SQ is site quality, and BA is stand basal area.

The effect of site productivity on survival is unclear. There is empirical evidence that in plantations,

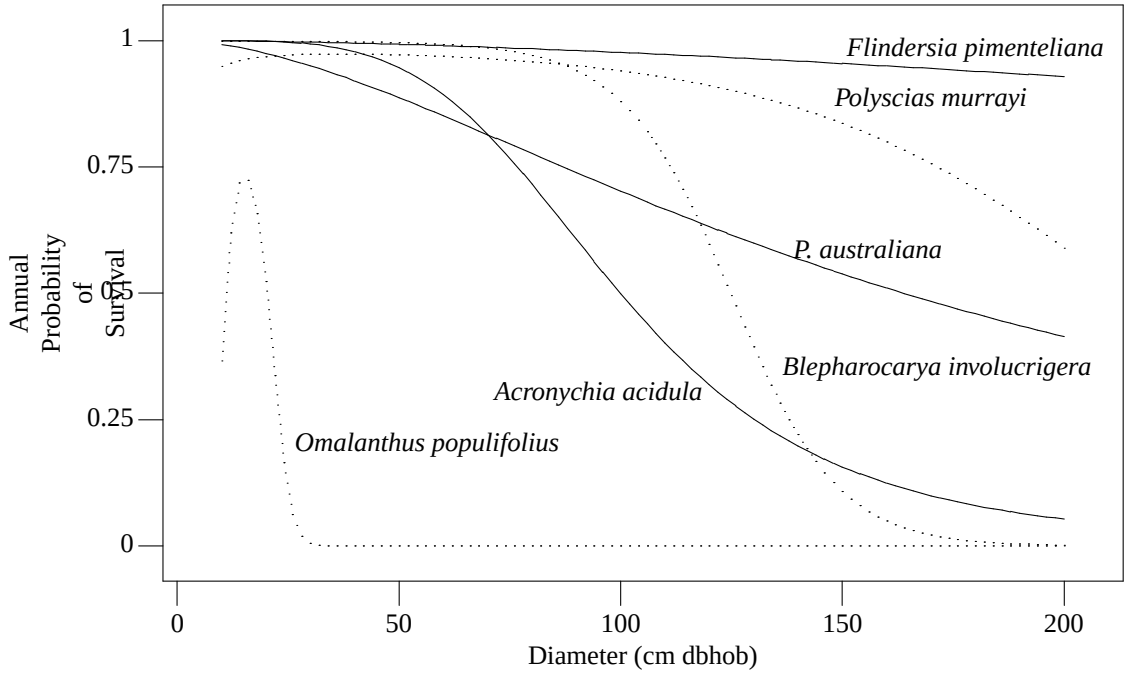

Figure 16. Predicted Survival Patterns for north Queensland TMF with Relative Size 0.5, Site Quality 7 and Stand Basal Area 30 m^2/ha

competition expresses itself earlier on better sites but it is not clear if this greater mortality is due to the higher basal area or due to the better site itself. Hamilton (1990) argued that sites with lower average potential diameter increment have lower mortality rates (i.e. better sites have higher mortality). However, it is logical to argue that better sites should be able to sustain a higher basal area, and that all other things being equal, better sites should have lower mortality. Vanclay (1991c) found site productivity significant and positively correlated with survival, but found no interaction with stand basal area.

Many models have employed past diameter increment to predict probability of mortality (e.g. Monserud 1976, Ek and Monserud 1979, Hann 1980, Hamilton 1986, Wykoff 1986, Wan Razali 1989). Buchman (1979) also included an intercept:

$$P = \beta_0 - \left(1 + e^{\beta_1 + \beta_2 DI^{\beta_3} + \beta_4 D}\right)^{-1}$$

so that survival was asymptotic to β_0 rather than to unity, to account to ''chance'' mortality amongst large vigorous trees. This model was later expanded to a seven parameter model to accommodate a more flexible response with tree size (Buchman 1983, Buchman *et al.* 1983).

Monserud (1976) illustrated that the use of predicted increments in mortality functions leads to different parameter estimates and a worse fit than the actual increments. Monserud found a function that could predict survival of all species in mixed northern hardwoods stands:

$$P = (1 + e^{-1.45 - 0.088D - 0.62\ \hat{D}I + 0.0015CI})^{-t}$$

where P was the probability of survival over a t year period, D is diameter, $\hat{D}I$ is predicted diameter increment and CI is a competition index. This function correctly classified 88 percent of survivals and 35 percent of deaths. An analogue of this equation using actual rather than predicted diameter increment (with $50.0DI$ replacing $0.62\ \hat{D}I$) correctly classified 98 percent of deaths and 90 percent of survivals.

Using predicted rather than actual diameter increments results in a poorer fit to the data, and means that the

increment model cannot be revised without also revising the mortality model. If mortality can be estimated from predicted increments, then it should also be possible to estimate it from the explanatory variables for increment without the intermediate step of estimating diameter increment. Modelling mortality directly from tree and stand variables should be a more robust approach.

9.2.4 Computational Considerations

The traditional approach to developing a prediction equation is to linearize the model and to use ordinary linear regression. To adopt this approach, each explanatory variable is broken into a number of discrete classes, and the proportion of dead trees is computed for each class. For this approach to be valid, at least one dead and one live tree must be represented in every class included in the regression analysis. This can be very demanding of data, and enables only a few explanatory variables to be considered. The linear transformation also transforms the response surface, and this may result in parameter estimates based on incorrect assumptions about the errors.

A more efficient way to estimate parameters is to fit the model to the individual observations using weighted nonlinear least squares (Hamilton 1974, 1980) or using generalized linear regression (e.g. Aitkin *et al.* 1989). Although these techniques still require considerable amounts of data, especially when mortality rates are low, it is more efficient than the traditional approach using transformed group data.

Where the probability of mortality is computed for individual trees, it may be used in deterministic or stochastic models. In stochastic models, determining which trees die can be resolved using a Monte Carlo approach. In deterministic models, these probabilities can be interpreted as proportions, and the stocking in each cohort or class reduced by the predicted proportion. These alternative approaches should produce compatible predictions (Ek 1980, Weber *et al.* 1986). There are computational advantages in simulating mortality deterministically unless the user is specifically interested in studies of variability.

9.3 Catastrophic Mortality

Catastrophic mortality is generally ignored in mortality functions. If it is taken into account, yields may be predicted in the absence of catastrophic mortality, and the final estimate may be reduced by an arbitrary allowance to account for such losses. There may be good cause for adopting this approach, as any attempt to include catastrophic mortality data in regression analyses may severely compromise the assumptions of normality, and result in biased estimates. However, an objective estimate of the reduction to apply to the final yield is preferable to a subjective guess.

Hamilton (1980) suggested that catastrophic mortality should be modelled in two stages: firstly the probability of a catastrophe should be predicted, and then a conditional function should be used to predict the probability of mortality given that a catastrophe has occurred. This two stage approach may be used to determine a weighted estimate of annual mortality for a deterministic model which accounts for catastrophic mortality (Hamilton 1980), or may be implemented directly in a stochastic model.

Several models simulate the interacting effects of pest or disease populations and stand condition. Stage (1973) explicitly modelled mortality due to mountain pine beetle using a deterministic model incorporating tree and stand characteristics (phloem thickness, bole surface area. stand density, etc.) and beetle population. Similar models exist for gypsy moth (Valentine and Campbell 1975), fusiform rust (Arvanitis 1978), spruce budworm (Fisher 1980), white pine blister rust (McDonald *et al.* 1980), and Douglas-fir tussock moth (Monserud and Crookston 1982). Reed (1980) considered the development of forests after the catastrophic elimination of one of its component species.

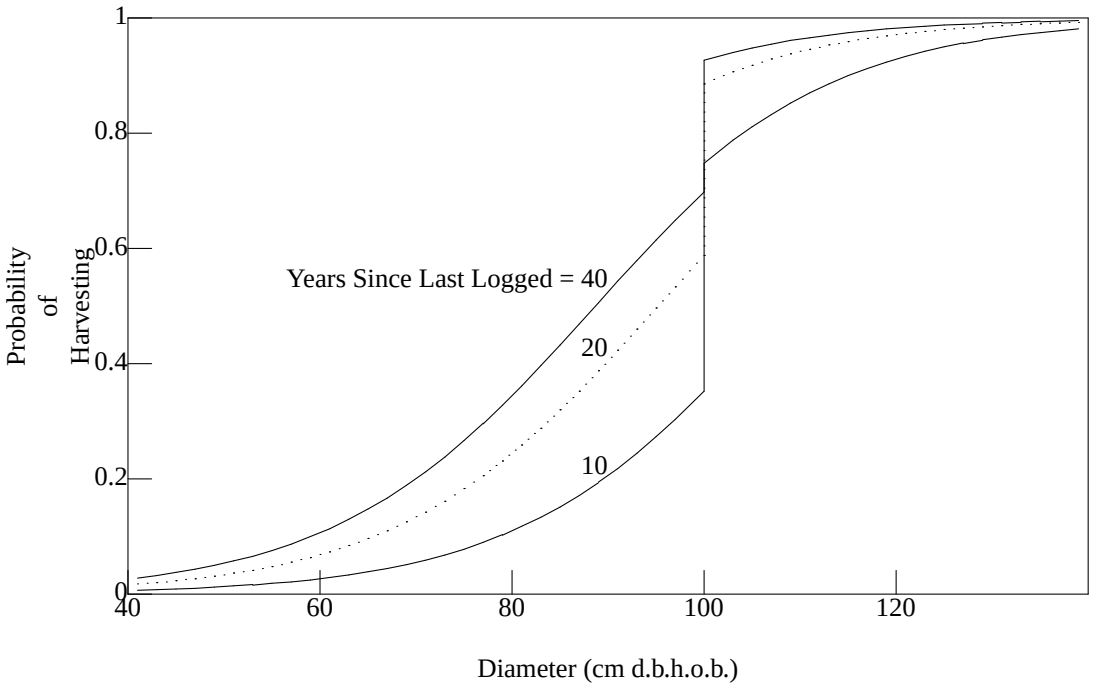

Figure 17. Harvesting Probabilities for *Cardwellia sublimis* (Vanclay 1989b)

9.4 Management Induced Mortality

9.4.1 Harvesting

Models for even-aged stands frequently use equations to predict the diameter distribution of stems removed in thinnings, which are typically systematic or ''from below''. Clearfelling of the final crop is even easier to model, in both plantations and natural forests.

In uneven-aged stands, there may be a greater variation in the stand condition, and selection logging tends to remove the largest stems, often according to defined size limits. Thus functions to predict the distribution of removals are likely to be of little use, and a better approach is to specify ''cutting rules''. Brand (1979, 1981b) gave examples of cutting rules formulated through decision trees. The first step in this process is to formulate the objectives and assumptions implicit in the management of the forest type under consideration. Then the decision tree can be formulated, and explicit cutting and treatment rules formulated for each branch of the tree.

Cutting rules may take three forms: (1) specifying the number of trees in each size class to be removed at the time of logging. (2) specifying the percentage of trees in each size class to be removed, or (3) specifying the number of trees in each size class to be retained after logging. The first of these reflects a market oriented approach which may be impractical to implement (i.e. cannot remove more trees than exist). The third option is ideal for plantations where the desired residual stocking can be specified; in TMF the composition and stocking may be so variable that this option may be impractical. Thus for TMF, the second approach is often the only viable alternative, and is of practical use in computer simulation studies (e.g. Hahn and Brand 1979) and as a field guide for officers marking trees for removal.

Vanclay (1989b) presented a quantitative model to predict the stand fraction removed in harvesting. He used logistic functions to predict the probability of removing any tree, as a function of tree species and size, and time since last harvest. The equation for some of the prime veneer species (e.g. *Cardwellia*

sublimis) was (Figure 17):

$$P = (1 + e^{6.088 - 0.07411DBH + 19.3/TSL - 1.696CL})^{-1}$$

where P is the probability of harvesting, DBH is tree diameter (cm d.b.h. or above buttress), TSL is years since last harvest and CL is a dummy variable which takes the value one if the tree exceeds the cutting limit ($DBH > 100$) and zero otherwise. This harvesting simulation model was used in yield studies of TMF in north Queensland (Vanclay and Preston 1989).

9.4.2 Silvicultural Treatment

In a growth model, treatment (e.g. liberation thinning, timber stand improvement, etc.) of stands can be considered in much the same way as logging. Treatment rules may dictate the removal of all old stags remaining after logging, a reduction in stocking in the smallest size classes to some specified amount, and/or poisoning of non-commercial species. Such treatment can be implemented in the model in the same way as harvesting. Treatment (including climber cutting) may induce a response greater that that due to a reduction in stand basal area. Vanclay (1988a, 1991b) found a transient stimulus to diameter increment (above that due to the reduction in stand basal area) due to treatment, and modified his diameter increment functions to accommodate this.

9.4.3 Logging Damage

Damage to the stand resulting in death of trees at the time of logging should be considered, and if significant, should be quantified in the model. This component should not be included as regular mortality, as its impact is dependent on the frequency of logging. This may be particularly significant if the model is to be used to determine optimal stand condition and optimum cutting cycle, when failure to identify this component separately may result in a biased estimate of the optimum cutting cycle.

Vanclay (1989b) predicted the incidence of destruction of trees in the residual stand as a logistic function employing tree size, topographic slope and basal area removed in logging:

$$P = (1 + e^{3.990 + 0.05958DBH - 9.689RBA - 0.05648SLOPE})^{-1}$$

where P is the probability that a tree of diameter $DBH(cm)$ will be destroyed in a harvesting operation which removes a proportion RBA of the standing basal area, and where $SLOPE$ is the topographic slope in degrees. Canopy height could also be a relevant variable to investigate.

9.4.4 Indirectly Induced Mortality

Logging may indirectly cause mortality in forest stands through a variety of causes, including the presence of logging waste or changes in the canopy conditions which may favour pests or diseases, or simply through the slow demise of trees mechanically injured during logging. Walters *et al.* (1982) report that mortality may treble for several years in a stand after logging, due to injury inflicted at the time of logging. A similar trend is evident in growth and yield studies in Malaysian dipterocarp forests. There are two possible ways of dealing with this situation. The first is to try to identify mortality arising from such causes, and fit a model to this data.

An alternative and probably superior approach is to include these deaths in the composite model, and to include terms in the function to account for time since logging. Hann (1980) used a measure of time since logging in his logistic function for composite mortality, and this described mortality during the few years after logging when it was considerably higher than in the absence of logging. However, Vanclay (1991c) examined time since logging when constructing his mortality functions for Queensland TMF, but found it not significant. Hamilton (*pers. comm.*) also found that time since logging and type of thinning had no impact on mortality rates following thinning in temperate forests. However, if any such mortality does occur, it should be distinguished from natural mortality for yield studies as it may influence the length of the optimal cutting cycle.

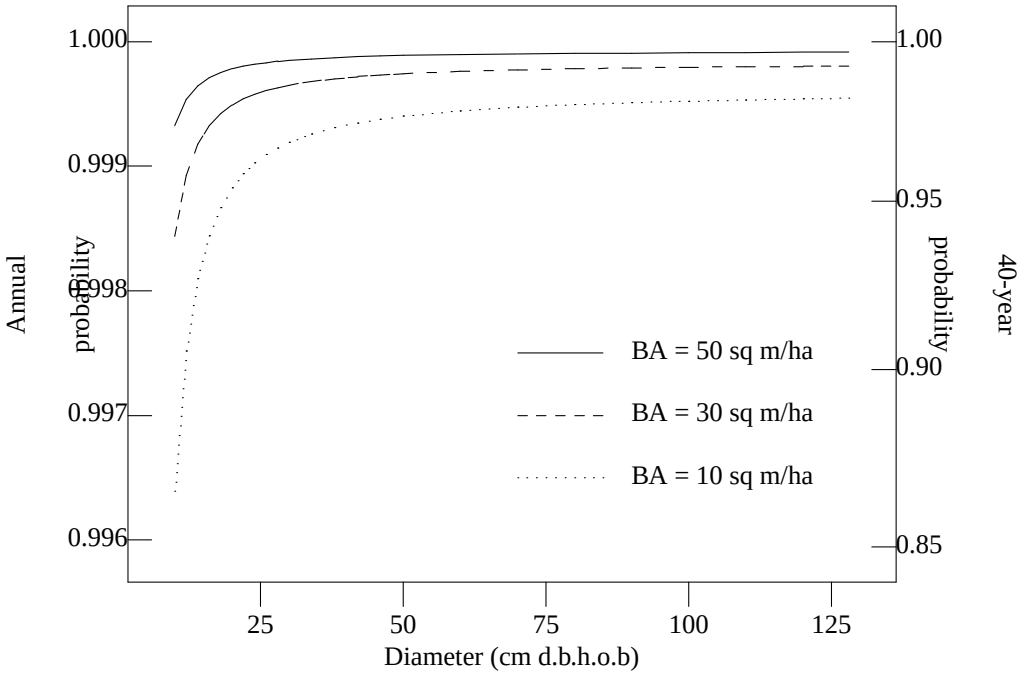

Figure 18. Proportion of Stems remaining Merchantable (Vanclay 1991a)

9.5 Merchantability

Whilst merchantability assessment may seem unrelated to mortality prediction, for modelling they pose identical problems. Predicting the rate of deterioration of useful trees is computationally identical to predicting mortality, and estimating the proportion of useful trees among those felled is the same as predicting the proportion to be felled.

9.5.1 Deterioration of Living Trees

Trees assessed as merchantable at time of inventory may not remain merchantable until the next harvest. Some of these once-merchantable trees may deteriorate to the extent that they are no longer of commercial importance, and this deterioration should be taken into account. Although small, this deterioration is cumulative and becomes sufficiently large during a cutting cycle to warrant inclusion in yield studies.

Vanclay (1991a) predicted deterioration as a logistic function of stand basal area, tree size, time since logging and soil type. For the more durable and valuable species (e.g. *Toona australis*), time since logging was not significant, and the prediction equation was (Figure 18):

$$P = (1 + e^{-7.450 - 0.04195BA + 22.49/DBH + 0.4213CG})^{-1}$$

where P is the annual probability that a merchantable tree remains merchantable, BA is stand basal area (m^2/ha) and CG is a dummy variable which takes the value one on coarse granite derived soils and zero elsewhere.

Hann (1980) studied an analogous problem. Young vigorous *Pinus ponderosa* trees have dark-coloured bark, while the mature slow-growing trees develop a yellow-coloured bark. Hann found that trees in these two categories displayed different growth and mortality rates, and obtained improved predictions by modelling them separately. However, this meant that it was necessary to predict the conversion from

blackjack to yellow pine. One problem is that this conversion, like deterioration, is a slow but continual process, and the classification of a tree as one or the other is extremely subjective. Hann screened several measures of site productivity, stand density and tree size, and his final model was

$$P = \left(1 + e^{\beta_0+\beta_1 D+\beta_2 D^2+\beta_3 SI}\right)^{-1}$$

where P is the proportion of blackjack (vigorous) trees converting to yellow pine, D is diameter and SI is site index.

9.5.2 Merchantability of Logged Stems

Not all trees assessed as merchantable and felled during harvesting will yield a merchantable log; some will be found, after felling, to be unmerchantable, and this must be taken into account in yield studies. The harvesting model could treat these stems as logging damage, in which case the predicted harvest should comprise only the merchantable stems. An alternative approach is to use three functions for modelling harvesting: one to predict all felled stems, one to predict the destruction of trees in the residual stand, and one to predict the proportion of felled stems which are merchantable. This latter approach is statistically more efficient (e.g. Hamilton and Brickell 1983).

The method of predicting merchantability must be comparable with the assumptions implicit in the derivation of volume equations. Vanclay and Preston (1989) employed volume equations which predicted net log volume per tree, given that the tree contained at least one merchantable log. An alternative approach is to estimate defective volume and subtract this from estimates of gross tree volume. Vanclay (1989b) predicted the proportion of stems assessed as merchantable and which realized a commercial log when felled, whereas Strub *et al.* (1986) predicted the proportion of all trees in the stand which would be merchantable during clearfelling.

Vanclay (1989b) predicted this proportion of apparently merchantable stems which realize a merchantable log when felled, with a logistic function employing tree species and size (e.g. for cabinet and veneer species such as *Toona australis*):

$$P = \left(1 + e^{1.565+0.0129 DBH}\right)^{-1}$$

where P is the proportion of apparently merchantable trees which realize a commercial log and DBH is tree diameter (*cm* d.b.h.o.b.).

Strub *et al.* (1986) predicted merchantability of *Pinus taeda* with an exponential function employing tree diameter and mean height of dominant and codominant trees:

$$P = \frac{1 - e^{0.511(19-D)}}{1 + e^{9.09-0.751H}}$$

where D and H are tree diameter (*cm* d.b.h.o.b.) and mean height (*m*) of dominants respectively, and P is the proportion of the total stocking which is merchantable.

The approach of Strub *et al.* (1986) is applicable in even-aged stands which are to be clearfelled. In selectively-logged TMF, Vanclay's (1989b) approach may be more practical.

9.6 Conclusion

One reliable and efficient way to model mortality, merchantability and similar relationships is to fit logistic functions to the individual tree data using maximum likelihood estimation. Although these techniques are not always presented in elementary statistics texts, they are available in many statistical packages (e.g. Aitkin *et al.* 1989) and are not difficult to use. Such packages allow models to be fitted to the individual tree data enabling the most efficient use of these data and realistic assumptions regarding the error distribution.

10. Regeneration and Recruitment

Regeneration is often negligible in plantations and even-aged stands, and need not be simulated in growth models for these stands. Many models for uneven-aged forests also assume that recruitment will be negligible or will not influence short to medium term estimates of yield. However, yield studies for TMF require prediction of regeneration or recruitment, otherwise estimates will be biased and sustainability will be indeterminate.

Regeneration models predict the development of trees from seed or seedlings. As suitable data for modelling regeneration are difficult to obtain, many models predict recruitment rather than regeneration. *Recruitment models* predict the number of stems reaching or exceeding some specified nominal size limit (e.g. 1.3 m height, 3 m height, 10 cm d.b.h.o.b., etc.). Recruitment models may employ a static approach which predicts a constant amount each year irrespective of stand condition, or may be dynamic and respond to stand condition.

10.1 Static Recruitment Approaches

Static approaches to the prediction of recruitment assume that the amount of recruitment observed during the period of data collection reflects the long term average, and that this amount will not vary greatly between projection periods predicted by the model. Such assumptions of static recruitment are common in matrix approaches, but some matrix models employ more realistic approaches. In Usher's (1966) matrix model for Scots pine, the number of recruits increases as the number of trees in the larger size classes increases. Whilst such a relationship is easy to implement in a matrix model, it is not realistic and is unlikely to provide acceptable results. More realistic approaches are illustrated in Buongiorno and Michie's (1980) general matrix approach where recruitment was negatively correlated with stand density, and in Bosch's (1971) Leslie matrix which allowed regeneration to occur only after the death of another tree.

Grimes and Pegg (1979) predicted recruitment into the smallest class (20-30 cm d.b.h.) as a simple linear function of site index and basal area of stems greater than 7.5 metres high but less than 20 cm d.b.h. This reserve of small stems was assumed to remain constant throughout the projection period. Many models follow this approach and assume that stocking in the smallest class (or in the ''reserve'') remains constant (i.e. any upgrowth from this class is replaced by ingrowth) and is the source of all recruitment.

Belcher *et al.* (1982) predicted the regeneration entering the STEMS projection system 15 years after harvesting of the previous stand. Decision trees were subjectively compiled to enable the regenerating stand to be predicted from the original stand, management and site characteristics (e.g. moisture). A normal distribution of stem sizes was assumed.

Although these approaches are rather empirical, they may provide useful estimates of recruitment for stands which do not differ greatly from the source stands used for model development.

10.2 Dynamic Recruitment Models

Other approaches attempt to predict the number of stems recruited as a function of stand condition. These vary from the highly empirical to those that model some simple biological hypothesis. Letourneau (1979) used an empirical approach with 33 estimated parameters to predict numbers of stems, and accounted for time between remeasures in his size function. Landford and Cunia (1977) predicted total number of recruits (at 4 inches diameter) deterministically, but the size and species stochastically. Both these models used estimates of sapling density (numbers of stems in the 1, 2 and 3 inch d.b.h. size classes) in their equations and this may limit their utility for forecasting from operational inventory data.

Figure 19. Recruitment at 20 cm diameter in Queensland TMF (Vanclay 1989a)

Hann (1980) predicted recruitment using an exponential function containing the site index, stand basal area, and basal area in the smallest size class. Hann's simulation cycle was five years, and this ensured a realistic five-year lag in the appearance of recruitment stimulated by the reduction in stand basal area following logging.

Vanclay (1989a) predicted the total amount of recruitment at 20 cm diameter in Queensland TMF as a linear function of stand basal area and site quality:

$$N = 5.466 - 0.06469SBA + 1.013SQ$$

where N is number of recruits ($trees/ha/yr$), SBA is stand basal area (m^2/ha) and SQ is site quality. The composition of this recruitment was determined by predicting the proportion in each of five species groups, and standardizing the proportions. The proportion for each species groups was predicted from the stand basal area, the site quality and the basal area of that species group (e.g. for the large, fast-growing species):

$$P_1 = \left(1 + e^{2.407+0.005608BA-0.01105B_1-0.00464B_1SQ}\right)^{-1}$$

where BA is total stand basal area (m^2/ha), B_1 is the basal area of group 1 species (m^2/ha), and SQ is site quality. These proportions were then standardized to ensure they summed to unity (Figure 19):

$$P'_1 = \frac{P_1}{\sum_i P_i}$$

Botkin *et al.* (1972) adopted a more biological approach to modelling recruitment at 0.5 cm diameter on their ten metre square plots. They assumed that a seed source was available for each of the major species considered by their model, and compiled a list of possible species for the plot being modelled, on the basis of shade tolerance, growing season and soil moisture. If the plot leaf area index (LAI) was less than a specified threshold, 60 to 75 cherry trees were recruited on the plot. If the plot LAI exceeded the first threshold, but was less than a second larger threshold, some (0-13) birches were recruited. If the plot LAI exceeded both these thresholds, a random choice of the remaining suitable shade tolerant species was made, and a random number (0, 1 or 2) of each species was recruited. Botkin *et al.* (1972) simulated mortality of these seedlings to ensure that only two percent of fast growing and one percent of slow growing seedlings could reach the overstorey. Shugart and West (1977) followed a similar approach, but identified the requirements of each species for mineral soil or leaf litter, introduced stochastic elements of variable weather and animal browsing, and also modelled sprouting from dead trees. They recruited trees when they reached breast height. Similar succession models exist for subtropical rainforest in Australia (Shugart *et al.* 1980) and tropical rainforest in America (Doyle 1981).

Reed and Clark (1979) followed an approach somewhat similar to Botkin *et al.* (1972), but introduced alternate seed-years and ''off-years'' and imposed a maximum stocking of 2500 stems per hectare, irrespective of size, above which no recruitment could occur.

10.3 Two-stage Approaches

One of the difficulties in modelling recruitment is the great variability in regeneration. Stand condition accounts for some of this variation, periodicity of mast years and prevailing climate accounts for some, but regeneration remains a rather stochastic process, providing difficulties for efficient model estimation.

Much of the variability associated with regeneration is due to the fact that during any period some regeneration may or may not occur, and that if the data are partitioned into a two-state system, the ability to predict the amount of regeneration is greatly enhanced. With a two-state approach, the first equation estimates the probability that some regeneration or recruitment will occur, and can be estimated using logistic regression with presence/absence as the dependent variable (c.f. mortality functions). The second stage is a conditional function to predict the amount of recruitment, given that some is known to occur, and can be estimated using ordinary linear regression. Hamilton and Brickell (1983) gave an example of such a two-stage approach applied to the prediction of defective volume in standing trees, which can be applied

equally well to modelling recruitment.

Stage and Ferguson (1982) and Ferguson *et al.* (1986) used a two-stage approach to predict recruitment in the Prognosis model. They used a stochastic procedure to predict the regeneration on 50 subplots each 1/300 acre (about 0.001 ha), and these data were aggregated into the main Prognosis model at 10 and 20 years after disturbance (Prognosis has a 10-year simulation cycle). They predicted the probability that some regeneration would occur from environmental variables (habitat, slope, aspect, elevation), distance to seed source, residual basal area and time since disturbance. Given that regeneration is known to occur, the expected number of trees is determined using pseudo-random numbers, and determines the number of cohorts for that subplot. The number of species present, and the identity of these species were also stochastically determined. Height of regeneration is deterministically predicted. The model considers three categories of regeneration: ''best'' trees comprising the tallest two trees per plot regardless of species, the tallest tree of each additional species present, and the tallest four of any remaining trees. Best trees were considered as advance growth if established three years prior to disturbance, and would then be allocated a shade tolerant species. Regeneration within each category was predicted independently, and differed considerably (advance growth was more likely to be shade tolerant species).

Vanclay (1992) predicted the annual probability of any regeneration for each of 100 species, given the stand basal area, the presence of the species in the existing stand, and years since the last disturbance. This approach predicts probabilities independently for each species and contrasts with Ferguson *et al.*'s (1986) approach where the collective probability of regeneration was modelled, and the species occurrence predicted from the amount. Vanclay's function could be employed stochastically, or deterministically in which case the annual probabilities for each species are summed until the cumulative probability exceeds unity. There is some evidence to support such a deterministic approach; Webb *et al.* (1967) and West *et al.* (1988) found that soil moisture and fertility affected species occurrence in TMF, and Austin *et al.* (1984) and Moore *et al.* (1990) used deterministic algorithms to map vegetation types in mixed eucalypt forests in southern Australia. The amount of regeneration, given that it is known to occur, was predicted from stand basal area, site productivity and the relative abundance of the species in the stand (Vanclay 1992).

10.4 Regeneration Models

Some models simulate the growth of trees from seedlings to breast height within a separate regeneration model or ''understory operation'' (Ek and Monserud 1974a,b, Dudek and Ek 1980). The approach is sufficiently flexible that almost any size may be used as the criterion for recruitment into the main stand. Ek and Brodie (1975) simulated only suckers developing after logging (predicted from stand basal area before and after logging, site index and treatment), but many models simulate the development of regeneration throughout the development of the stand.

Modelling may start at any of a number of stages. Leak (1968) modelled regeneration from the stage of flower development, Ek and Monserud (1974a,b) from seed fall. Germination could equally well be considered, and Vanclay (1988a) started with ''establishment'' when the seedling has survived its first year after germination.

An advantage of the approach is that it realistically models the time taken for regeneration to be recruited following a reduction in stand density due to logging. Recruitment models which employ an expression of stand density may lead to overestimates of recruitment in the projection period immediately following logging. However, in TMF where there may be large reserves of advance growth, this may not be a problem.

Vanclay (1988a) predicted the amount of established one-year-old regeneration in *Callitris* forest using an exponential function of stand basal area and site productivity:

$$N = e^{0.4974SQ - 4.449 - 0.01723BASQ} - 1$$

where N is the number (per hectare) of one year old seedlings established, SQ is site quality and BA is stand basal area (m^2/ha). Regeneration was modelled as cohorts representing height classes until it reached breast height, when it was recruited to the main model. A maximum of ten cohorts were employed. Under ideal conditions (good sites with low stocking), these cohorts represented annual flushes of regeneration. Where growth of regeneration was slower and took more than ten years to reach breast height, the most similar cohorts were amalgamated to ensure that the limit of ten cohorts was not exceeded.

Ek and Monserud (1974a,b) adopted a more sophisticated approach to predict recruitment into their stochastic distance-dependent individual tree model. The regeneration model used cohorts representing the number of stems for each species and age in a number of subplots within the main plot being simulated. A Monte Carlo approach selected good, moderate and poor seed years according to the observed frequency for each species. Seed and sprout production were estimated for each overstorey tree as a function of its size and the threshold age, and were distributed across the subplots according to the parent tree's position, height and crown width. Germination was predicted as a stochastic function of microsite and canopy cover conditions. Each year, a germinant or tree in the understorey may die, or survive and attain some height increment (function of cover, species and age). When tree height reached breast height it was recruited into the main model. If a tree did not attain this height within a specified time (e.g. 25 years for black spruce), it died.

Monserud and Ek (1977) refined this approach, improving the efficiency by reducing the number of cohorts to be modelled. They assumed that understorey tree size was more relevant than tree age, and modelled the development of trees to 7.6 metres height using five height cohorts of varying size, using the movement ratio approach (see section 6.2). The height increment of the mean tree was predicted from the potential height increment (a function of height and site), overstorey competition index (a relative size-distance index), shade tolerance (a function of species and height), and stand density. Monserud and Ek determined that five cohorts were required to model the understorey without compromising accuracy. Such detailed approaches (Ek and Monserud 1974a,b, Monserud and Ek 1977) may not be warranted for yield prediction models, but may be relevant in models used to analyse silvicultural alternatives for intensively managed stands.

One difficulty with regeneration models is ensuring compatibility with inventory data when the model is used for yield prediction. Inventory data frequently sample only the larger stems (e.g. ≥10 cm d.b.h.), and smaller stems may remain unsampled. Thus there may be a ''gap'' in the data. Such problems are more common for regeneration models (which predict regeneration at very small sizes) than for recruitment models which predict recruits at larger diameters (e.g. 10 cm). To avoid this gap (or censorship), it is necessary either for inventory to provide a count of the smaller stems, or for a model to predict the likely incidence of such stems from overstorey stocking. Augmenting such censored data with an ''average'' small tree distribution for the forest type is preferable to using the unadjusted censored data (Randall *et al.* 1988).

10.5 Conclusion

Although regeneration models offer several desirable features, they are impractical for many TMF because of difficulty of species identification, absence of suitable data, and uncertainty of growth patterns. Many shade tolerant species may exist as advance growth for decades, until a gap appears in the canopy and provides an opportunity for these stems to grow into the overstorey. Regeneration models may be unnecessarily complex for deterministic yield prediction models in TMF, and the two-stage approach may be more suitable.

11. Validation and Calibration

Validation implies a demonstration of the validity of a model, but is something of a misnomer. An hypothesis or model can never be proved correct; it can only be proved incorrect. However, the failure of a number of attempts to disprove an hypothesis gives more credence to the hypothesis. This is the role of validation in growth modelling.

Calibration may take several meanings. In the context of growth modelling, it usually refers to parameter estimation. Although the initial fitting of a model to data is sometimes called ''calibration'', I use the term in the more restricted sense of finding an adjustment or ''fudge factor'' to improve predictions for a specific locality.

Both these activities require an operational growth model and are often seen as ''optional extras'' rather than as an integral part of growth model development.

11.1 Validation

Goulding (1979) suggested that validation involves three steps: (1) ensuring that the individual functions used in the model are appropriate, (2) verifying that the computer code is correct, and (3) comparing the model with the behaviour of the forest under a variety of conditions. These three steps should detect three types of error (Leary *et al.* 1979a): the equations may be inappropriate for describing the process, the manner of combining the equations may be at fault, or the numerical constants obtained in fitting the model may not govern the system.

The first step involves scrutinizing the component equations comprising the model. Many of the relevant points to consider have been discussed in Chapter 4. The equations should follow biological theory: tree diameter, stand height and stand basal area predictions should be constrained so that an upper asymptote is attained, and should show a logical relationship with site productivity and stand density. The standard statistical assumptions should be satisfied and residuals should be examined. This phase of validation is normally carried out before the component equations are collected to form the completed simulation model.

The second stage involves checking the computer program and the values of embedded constants to ensure that the model is correctly encoded. Some simple examples should be projected using hand calculations, and compared with the model's prediction. Model predictions can also be compared with forecasts from other independently-developed simulators. The model should also be tested on the data from which it was developed to reveal any basic flaws in its construction. The model should also be applied to hypothetical data corresponding to very sparse and very dense stands, sites of unusually low and high productivity, and for long durations to ensure that the model provide biologically reasonable predictions. Discrepancies may not be due to faults in the model (the hand calculations or modeller's expectations may be at fault), but they should be treated as such until careful checks have ensured the model error-free.

The final stage of the validation process compares model predictions with independent data. Van Horn (1971) defined this phase as ''the process of building an acceptable level of confidence that an inference about a simulated process is a correct or valid inference for the actual process'', and emphasized that the validity of the model cannot be divorced from the objectives for which the model was constructed. Validation is not so much concerned with the ''correctness'' of the model, but rather with the inferences that may be drawn from using the model. Since no model will behave exactly like the system it is modelling, it is pointless to try to prove the ''truth'' of a model. All models are imperfect; at best they are a simplification of complex biological and ecological processes. The objective of the validation process is not to accept or reject the model as true or false, but to assess the quality of its predictions (Holdaway and Brand 1983, 1986). Thus the critical question is not whether the model is valid, but whether it is *useful*. One test of a forecast ''is not whether it eventually turns out to be right but whether it generates enough confidence for it to be taken as the basis for action'' (Leslie 1980). Burk (1986) argued that validation is an evaluation of the usefulness of a model as a whole, in providing suitably reliable information for a specific

problem or type of problem with explicitly stated objectives. The validity of a model thus depends on the application objectives and the performance requirements. Validation should lead to a description of the model in terms of generality, realism, precision and sensitivity (Stout 1990):

Generality: To what forest situations is the model applicable? Is it applicable to a wide range of conditions, or only a narrow set of conditions?

Realism: For the forest conditions modelled, does the model provide realistic predictions? Are diameter distributions as would be expected? Is volume production realistic?

Precision: For the conditions predicted, does the model meet set standards for accuracy and precision?

Sensitivity: Is the model so constructed that the overall performance is not sensitive to unrealistically small differences in the size of coefficients? If the value of a coefficient is changed slightly, will it produce a large change in predicted outcomes?

11.1.1 Procedure

Popper (1972) argued that scientists make bold imaginative conjectures, universal statements from which a collection of consequences may be deduced. Some of these consequences may be amenable to experimental testing, and any of these consequences which are tested and found to be false falsifies the conjecture. A conjecture which survives repeated attempts to falsify it is corroborated, or not yet falsified, and may begin to assume the status of a scientific law. In order to construct a rigorous test, we require that the test data be drawn from permanent sample plots not used in the development of the model. This test will be more convincing if the validation data are drawn from a different population (e.g. another forest of the same type, a different series of experiments, or data collected by another agency).

Repeated testing until our growth model fails need not lead to complete rejection of the growth model. Rather, we can define a region within which the model behaves satisfactorily. Thus we may initially propose that a model is valid for all stands of a given species composition, irrespective of location or management regime. Repeated testing and failure under certain conditions may then lead us to specify less general conditions for which the model is valid. This not only indicates the conditions for which the model may be applied, but also indicates the areas warranting further research.

In interpreting the results of the validation tests, the user should also keep in mind that the decision to accept the null hypothesis does not mean that the model is correct, or that it is the best possible model. On the other hand, the decision to reject the null hypothesis does not necessarily mean that the model is not useful for practical purposes. If the null hypothesis is rejected, the question is where and how the model fails and what can be done to improve it (Reynolds *et al.* 1981).

Hann (1980) suggested a number of rules for validation. Validation should be made on a data set independent of those used to develop the simulator. Validation should occur on the basic attributes that the simulator produces, as aggregation may conceal inconsistencies. Thus validation should examine attributes of each class of a stand class model, and attributes of each tree in an individual tree model. Validation should examine short and long term responses, as some biases may take a long time to manifest themselves. Objective rather than subjective tests should be used to quantify validity. Monserud (1989) suggested that optimization studies provided a discriminating test of validity, as his optimizer seemed to be remarkably efficient at exploiting seemingly minor quirks in the Prognosis model to arrive at unrealistic solutions. Thus optimization studies coupled with expert validation may provide a good basis for validation.

Hann (1980) used several cycles of validation to identify faulty components in his model. His first cycle simulated only increments, and used actual mortality, logging and recruitment and thus resolved that increment prediction was satisfactory. The second cycle predicted increments and mortality, and employed actual logging and recruitment, and so forth, until the final test included all the predicted values.

Hyink (1990) suggested three types of growth validation:

- Single period validation which examines short-term performance by initializing the model with real data from new and independent PSP measurements and examining the performance of the model over one remeasurement period.

- First-to-last validation where the model is compared against PSP data spanning the whole history of the plot. Such procedures are more likely to detect subtle trends and errors in components such as logging rules.
- Expert validation where knowledgeable individuals who may have an intuitive feel for the dynamics of the forest, are asked to operate the simulation system over a wide range of conditions and to identify areas of questionable performance.

Newberry and Stage (1988) suggested that validation should define relevant data and criteria for model behaviour, should involve sensitivity analyses and statistical tests, and lead to one of four possible conclusions: (1) the model is adequate, (2) the model needs revision using existing data, (3) the data are inadequate to evaluate the model and new data are required, or (4) that the model is irrelevant.

11.1.2 Partitioning Data

Validation in its purest form requires setting aside of data or obtaining independent data for validation. The most convincing validation would use a set of data drawn from an independent population measured over a long period, but such data are rarely available. Growth modellers frequently are faced with the decision of having to partition a data set from a single population into two subsets, one for development, and the other for the validation of the model. Where ample data are available, this partitioning causes few problems. However, when data are scarce, there is a temptation to use all the available data for development, in an attempt to improve the model. Unfortunately, this diminishes the ability of the modeller to demonstrate the quality of his model, and ignores the role of falsification of hypotheses in science.

There are several statistical guidelines to use in the partitioning of the data set, but in practice the choice seems to be largely arbitrary. In validating a model, the modeller is generally placed in the position of wanting to accept the null hypothesis (that the model is valid), and must avoid the temptation to weaken the tests, by for example, reducing the number of data sets, or artificially increasing the variance of stochastically simulated observations. Snee (1977) observed that a half and half split generally seems to be the most popular method in other disciplines, but fewer data are usually used for validation of forest growth models. West (1981) reserved a random selection of one quarter of his data for validation studies. Shifley (1987) also used one quarter, but selected these systematically. Hann (1980) reserved one third of his even-aged data (six plots, one in each basal area class) and one fifth of his uneven-aged data (24 plots, eight in each of three basal area classes), both of which were selected by stratified random sampling. Data for validation need not be as extensive as data for calibration. If multiple silvicultural treatments are not being evaluated, ten to fifteen plots spread over the range of stand conditions of interest may suffice (Goulding 1979).

In view of the differing requirements of developmental and validation studies, some guidelines can be given for the partitioning of data. Plots established for long periods with regular remeasurement, particularly those remaining unlogged and without other disturbance, may prove useful as a discriminating test of validity. If data from a geographically independent area (preferably not an outlying extreme) can be reserved without compromising the range of site and stand conditions represented in the model, they could provide a convincing test for the validity of the model. Alternatively, data collected by an independent agency could provide a good basis for validation.

The outcome of validation tests can be influenced by the selection of data: ‘‘like’’ data will provide a more optimistic result than validation with ‘‘unlike’’ data from another population. Thus the most convincing demonstration of model validity can be made only if the validation data are in some sense unlike the development data. Since growth models are used to forecast future forest conditions, one way to split the data is on time, and for example, to use data collected before 1980 for development, and since 1980 for validation. It is desirable to have test data available to validate the model over the widest possible range of site and stand conditions possible, and that selection of such data should be free of bias. Snee (1977) discussed the DUPLEX data splitting algorithm. This algorithm splits the data so that the two sets cover approximately the same region and have similar statistical properties. The DUPLEX algorithm takes a list of candidate points, standardizes them and computes the Euclidean distance between all possible pairs. The two points furtherest apart are assigned to the estimation set, the pair with the next greatest separation are assigned to the validation set, and the remaining points are assigned in turn to the estimation and validation set, selected in order of the greatest distance from the data in the set (see sequence numbers in Figure 20).

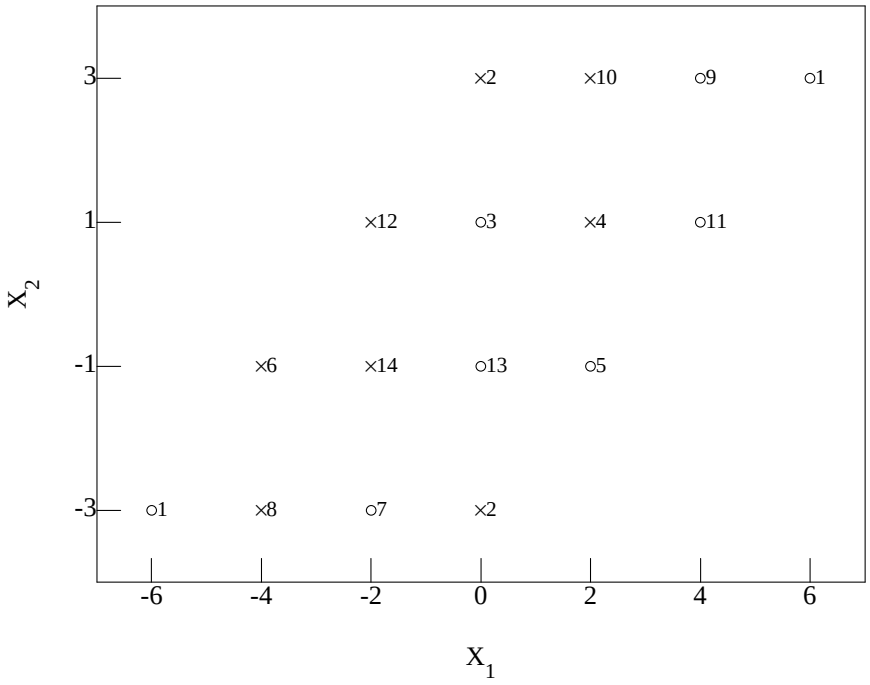

Figure 20. Example of Data Partitioned using DUPLEX Algorithm (Snee 1977) into Estimation (O) and Validation (×) Subsets. Numbers indicate sequence (see text).

This has the effect of creating two overlapping sets which cover different parts of the data space, thus providing an "unlike" set for rigorous validation. If the data contain replications or pseudo-replications, the candidate list should comprise clusters of data rather than individual points, otherwise the two tests will cover the same space.

The validation data should contain sufficient replications to enable the natural variability to be expressed. There are really two sources of error to be found in validating a model: model error and observed data variability (Burk 1986). Nature is not deterministic: two stands apparently with the same initial conditions may develop differently. Validation should employ sufficient data to provide a reasonable estimate of the expected actual stand condition.

11.1.3 Resampling Procedures

Suitable data for modelling are scarce, and researchers may be reluctant to set aside a sufficiently large amount of data for validation. An alternative is to mimic the use of independent data using resampling techniques such as cross-validation, bootstrapping and jackknifing.

Regression models may take the form:

$$\mathbf{Y} = g(\mathbf{X}, \beta) + error$$

where "*error*" is unknown, g may be non-linear and β are parameters to be estimated. *Apparent error* is computed by applying the fitted equation to the data used in fitting (e.g. r^2) and will normally give an optimistic view of the goodness of a model. *True error* is best estimated by fitting the model to independent data. However this is often not possible, especially when one considers how many independent data should be used. Resampling techniques provide an alternative.

Consider partitioning the data, with half for development and half for validation. If the resulting model failed to satisfy expectation, would it be improved by using all the data for development? But then there would be no independent test. Several re-sampling methods attempt to mimic independent validation

whilst enabling the full data set to be used for validation.

Half-splitting provides a solution, by fitting with half the data and validating with the remainder; the roles are then reversed. Each evaluation provides an estimate of true error which may be averaged. The entire data set may be used to obtain the final model and estimates of apparent error. This method provides pessimistic estimates of model performance but is computationally efficient (Burk 1990).

Cross-validation is the logical generalization of half-splitting. Rather than deleting half the data, each datum is deleted in turn and fitting carried out on the remaining $n - 1$ data. Validation is averaged from the individual deleted data. If the test statistic is squared error and the model is linear, the cross-validation estimate of true error is n times the PRESS statistic computed by many regression packages. The bootstrap and jackknife are similar to cross-validation, especially as sample size increases, but are computationally more complex (Efron and Gong 1983).

If liberal amounts of data are available to the modeller, some data should be relegated to an independent check of the model. In data-poor situations, it would seem irresponsible of the modeller not, at a minimum, to consider applying a re-sampling procedure. Burk (1990) found that half-splitting provided a pessimistic evaluation, and suggested the use of cross-validation and jackknifing, coupled with tests based on limited independent data.

One shortcoming of any resampling procedure lies in its dependence on the data. If the sample obtained does not do a good job of representing the variabilities and idiosyncrasies of the population of interest, the resampling procedure is doomed to failure. ''What's worse is those are just the circumstances under which the model itself should come under heaviest criticism'' (Burk 1990). However, this caveat also applies to independent data for validation.

11.1.4 Objective Tests

In order to provide an impartial assessment of the validity of a model, it is desirable to use objective ''statistical'' tests. Regrettably, there is no one test which can pass, fail or indicate the quality of a model, as its utility depends on your purpose. Goodness-of-fit statistics do not constitute a validation. One way to examine model performance is to plot the predicted and observed values of several tree and stand variables and see how they compare, how they change over time and how they are affected by other stand variables. Such plots are interpreted visually, and appraisals of performance are necessarily subjective.

Two simple criteria provide nearly all the information necessary in validation (Snee 1977, Burk 1986). These criteria are the model bias ($\sum(predicted - observed)/N$) and the mean absolute difference ($\sum|predicted - observed|/N$, where N is the number of observations used in validation). These formulae apply equally well whether validation is at the stand level or for a particular size or product breakdown. The model bias measures the expected error when several observations are to be combined by totalling or averaging. The mean absolute difference or accuracy measures the average error associated with the prediction of any one observation. Reynolds *et al.* (1988) suggested $\sum W(P - O)$ where W is a weight (e.g. basal area, volume, value) and P and O are the predictions and realizations respectively. They were concerned specifically with the special problems of validating diameter distribution models and offered a number of suggestions for this class of model.

Ek and Monserud (1979) established six tests for validation and comparison of two growth models:

1. paired t-tests for basal area and number of stems per hectare,
2. Freese's (1960) chi-square test for accuracy of the predicted values,
3. a Kolmogorov-Smirnov test to compare actual and predicted distributions of stand variables from several plots,
4. a simple linear regression of predicted on observed stand attributes (should have zero intercept and a slope of one),
5. regressions of residuals about predictions on observed initial stand variables, and
6. a Kolmogorov-Smirnov test to compare the diameter distributions.

Reynolds (1984) discussed several alternatives for validating a growth model. He pointed out that two different philosophies to validation can be identified. One approach uses statistical hypothesis testing and the other uses statistical estimation. ''Hypothesis testing procedures are appropriate when the question is whether a model behaves like the real system or whether a model meets a specified accuracy requirement of

the user. In other situations, however, there is no particular standard for comparison and the objective is simply to give the user of the model some estimate of how far predictions from the model will be from actual values''. Reynolds (1984, Reynolds and Chung 1986) presented formulae to calculate critical errors which provide confidence intervals for the expected magnitude of prediction errors. ''Prediction intervals and tolerance intervals are based on the estimation approach and should be very useful for indicating the size of the error when the model is used for prediction. A prediction interval is appropriate when the user is concerned with a single future prediction and a tolerance interval is appropriate when the user is concerned with the population of errors from a large number of predictions'' (Reynolds 1984). Error distributions of validations especially from stand class models, can be extremely long-tailed, and nonparametric intervals should be used in such cases.

11.1.5 Sensitivity Analyses

Growth models may contain many assumptions (e.g. simulated plot size, maximum number of tree records, record doubling or tripling, attenuation of calibration factor, etc.) which are not supported by data, may influence predicted outcomes and which may not be easily validated. If validation reveals that the model gives acceptable predictions, then these assumptions are probably reasonable. However, users of the model may wish to vary these parameters (e.g. to improve computational efficiency during complex calculations), so validation should include sensitivity tests to ensure that predictions are not unduly influenced by these assumptions. The effects of these assumptions should be checked by varying them and observing the effects on predicted yields. If sensitivity analyses reveal that one or more of these parameters has undue influence on predictions, the fact should be documented and revision of the model should be contemplated.

11.2 Calibration

Calibration implies adjusting a growth model so that it provides good predictions for a new population. This may entail estimating new parameters for some or all of the equations in the model, or may merely involve estimating a ''fudge factor'' to scale predictions.

STEMS (Belcher *et al.* 1982) is one growth model which has been ''transplanted'' to several other regions. Most copies have retained all the computer code and retained the form of all equations. Some have estimated new coefficients for all or most equations (e.g. Shifley 1987), whilst others have developed ''fudge factors'' to scale existing equations (e.g. Holdaway 1985, Smith 1983). Such fudge factors may comprise a single correction factor for each species, or may be correlated with some tree or stand variables (e.g. tree diameter or stand basal area). Smith (1983) applied three different fudge factors for each species, for each of three diameter classes. This means that the diameter increment function effectively becomes a discontinuous step-function and may be undesirable for some applications. Both cases require much the same procedure as is followed for initial growth model development. All equations used should be appropriate for the new location. Residuals should be inspected to ensure that the models are sufficient and adequate.

Calibration is not a panacea, even for existing ''good'' models. Attempts to calibrate STEMS to Australian forests using a single ''fudge factor'' (e.g. Swain and Turner 1988 for *Eucalyptus marginata* forest) or by re-estimating coefficients in component equations (e.g. Goodwin 1988 for mixed eucalypt forest) have been fraught with difficulty and results to date have been poor.

In calibrating a model for a new locality, two steps are involved. Firstly to validate the model using data from the new locality to determine if any calibration ''fudge factor'' is needed. Given that validation studies indicate some adjustment is necessary, the residuals about predictions should be examined to see if a single ''fudge factor'' would be adequate, or if a more sophisticated adjustment is necessary. If inspection of residuals indicates that a simple adjustment to increment rates would provide satisfactory predictions (e.g. analogous to a better site productivity), then such calibration should be attempted. However, if a more complex adjustment to growth patterns is indicated, it may be preferable to abandon calibration attempts and to estimate new parameters for all coefficients in the model.

11.2.1 Self-Calibration

The concept of self-calibration appears to have been first applied by Stage (1973), who used a single, simple correction factor based on increment cores taken on temporary inventory plots to adjust estimates of diameter increment. The method was formalized statistically by Burk and Scott (1976) and Meng *et al.* (1990). Stage (1981) discussed the rationale for, and implementation of, his approach, and discussed the attenuation of the adjustment over time. He calculated the deviation (observed − predicted) in the logarithm of the tree basal area increment, and adds this to the logarithm of his diameter increment function. He, in effect, assumed that his function has the correct ‘‘shape’’, and that only the rate of growth needs to be adjusted using a single parameter. This robust approach may be advisable if estimates of site productivity are unattainable, or if the climate varies greatly between the areas from which the development data were obtained and where it is intended to apply the model. Stage (1973) argued that the approach accommodates the model to ‘‘local peculiarities of site quality, genetic character and tree vigour’’, but cautions that ‘‘growth functions should be based on data derived from the area to which the model is to be applied; the self-calibration feature ... only partially mitigates that admonition’’.

The self-calibration feature in Prognosis has been implemented so that local adjustments are attenuated over time, and predictions gradually revert towards that of the standard uncalibrated model. Attenuation is modelled using an exponential decay function (Wykoff 1986) with the asymptote midway between 1.0 and the calculated calibration multiplier. The decay rate is such that it takes 25 years to reach midway between the calculated multiplier and the asymptote.

A contrasting application is the GROPE model of Alder *et al.* (1977) which was to have all its parameters estimated ‘‘automatically’’ by self-calibration from site-specific data. Such an empirical approach must ensure that all equations used are inherently robust, and that the data available for each site are extensive and sound. Although this is intuitively possible, effective implementation would require an expert system embodying many concepts of statistics, ecology and silviculture.

11.2.2 Validating a Calibrated Model

Validation of a calibrated model poses a special problem. In Stage’s (1973, 1981) approach, the model can be thoroughly validated prior to calibration. After calibration for some site, only two factors must be considered in validating the calibration - (1) the assumption that the ‘‘shape’’ of the basic functions holds is correct, and (2) that the calibration process has in fact removed the bias evident in the model prior to calibration. Whether or not the basic functions hold for a large range of sites depends largely on the model, and the specific functions themselves. Stage (1973, 1981) calibrated only the diameter increment function, and assumed that the other functions (height increment, bark ratio, crown ratio and mortality) would remain unaffected by ‘‘site quality, genetic factors and tree vigour’’.

The GROPE model (Alder *et al.* 1977) provides an option for data to be reserved for independent validation of the model, and this is clearly required in an empirical approach of this nature. The ‘‘model fitting cycle’’ of their model may be viewed as the normal development phase of model construction, and before output from the ‘‘simulation cycle’’ and ‘‘output cycle’’ of the model are employed for applications, the model should be validated with independent data.

11.3 Conclusion

If growth modelling pretends to be a science rather than an art, the models proposed should be falsifiable. That is, they must be able to be rejected through the normal process of validation. Validation must be viewed as an important phase of the development of the model, not only indicating the nature of the forests for which the model may be expected to yield reliable results, but also indicating areas of forest management which require further research. The temptation to use all the available data for the development of the model must be avoided, as it is equally important to have an independent set of data available for validation. The need for validation is not diminished through the use of ‘‘self-calibrating’’ models.

12. Application and Use of Growth Models

There is little point in developing a growth model unless it is to be used. Although model development may reveal some implications for forest management, the greatest benefit will accrue if forest managers use the model to investigate forest management alternatives. Accordingly, the model should be available, adequately documented and integrated into other information systems used by forest managers.

12.1 Resource Data for Simulation Studies

Considerable effort is expended in developing reliable growth models, but good predictions can only be made if the input data are also reliable. Thus users of growth models should take commensurate care in collecting the necessary input data. It is important that sampling be efficient and unbiased, and this requires decisions on stratification, plot size and trees to be measured. These topics are discussed in most inventory texts (e.g. Avery and Burkhart 1983), and the following is only a brief overview.

12.1.1 Stratification

''Precision is gained by dividing the population into as many blocks as expedient, even though the number of random sampling units taken from each is the minimum of two'' (Schumacher and Chapman 1954:61). Gains in precision are greatest when the proportion of the population within each stratum is known and when the within-stratum variation is small compared with the between-stratum variation (Cochran 1977). Smith and Burkhart (1984) found that stratifying by both site index and stocking improved the precision of volume estimates by 2/3 over simple random samples. Site index was the more useful variable, and sample size had no effect on the relative gains achieved through stratified sampling.

Mowrer (1989, Mowrer and Frayer 1986) discussed the effects of errors in initial conditions, and suggested that errors in applications data could have a greater effect on final levels of variability than the contributions from the growth model.

12.1.2 Plot Size

Plots should be of an appropriate size to ensure that results obtained are realistic, and are not an artifact of sampling. There are two issues: the size of the plot measured in the field, and the size simulated on the computer; the two need not be the same.

Upper and lower limits on plot size may be dictated by the need for plots to be homogeneous and representative of the forest stand. Zeide (1980) suggested that the optimum plot size for resource inventory is ''that which minimizes the total time required for location and measurement for a stated accuracy in the desired variable''. Husch (1971:46) suggested that plots should contain about twenty trees of measurable size. Hann (1980) considered 32-hectare plots which were divided into one-hectare sub-plots, and found that yield estimates from the subplots were more accurate than whole plot predictions. In TMF, one hectare plots may be impractical, and West *et al.* (1988) found 0.5 hectares to be the practical upper limit for homogeneous plots in north Queensland rainforests.

Plot size may have a considerable influence on results from individual tree models, but models are usually less sensitive to simulated plot size. In some models, there may be significant interaction between plot size and the regeneration sub-model (e.g. the gap-phase regeneration sub-model of Botkin *et al.* 1972). Vanclay (1991d) designed a tree list model which allowed the user to vary the plot size for simulation, and compared predictions from the same input data with different simulated plot sizes. For this model, differences tended to be small for deterministic predictions, but plot size had a major influence on the

variance of stochastic predictions.

12.1.3 Censored Data

It is important that inventory data be compatible with the data requirements of the model. Incompatibilities may arise most frequently in the measurement of small trees. For example, the growth model may predict recruitment at 10 cm d.b.h. and require data for all trees in the stand exceeding 10 cm diameter. However, some inventory data may only record trees exceeding 20 cm diameter, and such data is known as censored data. Another form of censorship is the failure to record non-commercial species during inventory. Censored data may lead to biased estimates of growth and yield. Randall *et al.* (1988) investigated the impact of censored data (omission of trees<13 cm d.b.h.) on yield predictions, and recommended that such data be augmented by ‘‘average’’ small tree distributions for that forest type.

12.2 Integrating Models into Information Systems

If an organization is to extract value from its growth models, efforts should be made to make the models available to forest managers and other interested staff. Growth models should not be reserved for use of head office staff only. Field managers should be able to use the models themselves, preferably in a ‘‘hands-on’’ interactive environment. The growth model should be viewed as a tool to assist in more informed management of the forest, but must not threaten the experience and judgement of the manager. The model must be easy to use, well documented and readily available.

Vanclay (1990) described how a growth model could be integrated into an inventory reporting system so that plots can be reported as at date of measure or as simulated to any future date, and reported as individual plots or summarized into stratum or forest estate reports. Moore and Lockwood (1990) described a yield prediction system linked to a geographic information system, and Pelkki and Rose (1988) describe the integration of such a system with an expert system to enable automatic production of stand management prescriptions. Vanclay (1990) also stressed that the model should also be available in a flexible and easy-to-use yield forecasting system, and as an interactive package to enable investigation of ‘‘what if ...’’ questions. Interactive simulation options provided in this system included

- Grow the stand a specified number of years, or until the stocking (stand basal area, standing or merchantable volume or average stem size) reaches a specified level.
- Log the stand, using a logging simulator, removing all merchantable stems exceeding a specified size, or soliciting the user’s intention for each species and size class specified by the user.
- Treat the stand, applying standard treatment rules, reducing the stocking of specified species and sizes to a specified residual, or soliciting user’s intention for each species and size class.
- Undo the last (grow, log or treat) command.
- Save the present stand for future reference. This is useful for examining different logging or treatment options for a stand which can be logged, treated and grown several times, using different strategies.
- Restore the stand as at last save.
- Display the current stand as a stand table, using size classes specified by the user.

A macro facility is also provided, so that silviculture can be defined in a series of instructions, and repeatedly invoked so that the long term consequences can be investigated. Such studies can be done individually for single stands, or simultaneously for a series of stands. Whilst such studies may provide useful results, they should always be investigated further in field trials.

12.3 Optimization Studies

Optimal forest management decisions may require the following information (Hann and Bare 1979):

- optimal sustainable diameter distribution including maximum tree size and stocking by diameter and species classes,
- optimal cutting cycle and conversion strategy, and
- optimal schedule of treatments for all stands to meet forest-wide objectives and constraints.

Such information may be obtained from analyses at the stand level, or at the forest level. Stand level analyses (e.g. of optimal sustainable diameter distribution, cutting cycle or conversion strategy) are relatively easy to undertake, but cannot indicate how to implement such a strategy for a forest estate. For instance, the need to maintain a steady supply of timber may preclude the immediate adoption of an optimal cutting cycle in all stands in the forest estate. Stand level analyses can indicate the optimal strategy for a single stand in isolation, but an optimal schedule for all stands requires forest-level optimization.

Some techniques relevant to TMF are discussed here, whilst more comprehensive reviews of optimization studies and methodology for uneven-aged forests were given by Bare and Opalach (1987) and Haight (1987, Haight and Monserud 1990a,b). Haight (1985) demonstrated that static and dynamic studies may not provide the same results; careful consideration of the assumptions implicit in each approach is necessary to determine the appropriate result.

12.3.1 Static Studies

Useful information on stand structure and tree size for optimal production can be made from simple analyses of the component equations comprising the growth model. Such studies may use a diameter increment function and a volume equation to determine the current and mean annual volume increment for a single tree, and thus determine the optimal size for harvesting (i.e. the size at which the MAI is maximum, if volume is to be optimized). Such studies generally assume that stand basal area and the relative competitive status of the tree remain constant, and are thus termed static studies. Dynamic studies simulate the stand dynamics and provide a more accurate picture, but require additional resources.

Vanclay (1989a) reported some simple static studies of optimal tree size for harvesting, and illustrated how the optimum changes with site productivity and stand basal area. These studies generally supported the existing harvesting guidelines (Preston and Vanclay 1988), although some limits specified in the existing guidelines were apparently smaller than optimal (Table 6). The guidelines specified that trees exceeding the cutting limit should be harvested unless they are of outstanding vigour and form, and that all trees exceeding the ''retention'' diameter should be harvested, irrespective of vigour and form, unless required as seed trees. Vanclay (1989a) estimated optimal diameters for these cutting and retention limits by calculating the diameter at which the mean annual volume increment of individual trees reached its maximum, assuming no mortality for the retention limit and average mortality losses for the cutting limit (Table 6). The incidence of internal stem defect was not considered, but would slightly reduce the optimal diameters for some species. Such an analysis would over-estimate diameter limits, as in practice harvesting would occur approximately every 40 years, not once for every centimetre of diameter growth.

TABLE 6. Diameter at which Tree Volume Growth is Maximized (Vanclay 1989a)

Species Group	Diameter (cm) at which to harvest tree (assuming coarse granite soil)					
	Results from Static Optimization Study				Treemarking Guidelines	
	Retention	Cutting Limit if Basal Area is:			(Preston and Vanclay 1988)	
	Limit	20 m^2/ha	30 m^2/ha	40 m^2/ha	Retention	Cutting
Large, fast-growing	128	105	101	96	80–100	60–100
Large, slow-growing	143	108	105	101	50–100	50– 80
Small, fast-growing	109	71	64	54	50– 90	50– 70
Small, slow-growing	101	66	60	53	50– 90	50– 70

12.3.2 Dynamic Studies

Dynamic optimization studies are more complex, but provide a more realistic analysis of stand dynamics. The growth model's ability to predict yields corresponding to various stand conditions can be exploited to compute the stand condition which maximizes some benefit which may be log volume, sawn volume, discounted net revenue or some other criterion, subject to several constraints. Although many constraints can be accommodated, a single objective function is required and may prove something of a limitation. If it is desired to maximize several products, these must be converted to common units (e.g. dollars or cubic metre equivalents) and combined into a single objective function. Some agencies may not be profit motivated, and non-timber benefits may be difficult to quantify. Thus it may be desirable to maximize sawn volume, for example. However, Adams (1976) illustrated that the use of any criteria other than revenue will lead to a lower revenue accruing to these stands. Whilst most studies optimize the most likely outcome from the forest using a deterministic model, or the mean or median of a stochastic model, there may be merit in maximizing the lower confidence limit (e.g. Valsta 1991) instead of the most likely outcome, particularly in view of the nature of the long term commitments made by forest owners. Most research effort has focused on finding the diameter distribution which maximizes revenue (or some other criterion), but a few studies (e.g. Valsta 1988) have examined the question of species composition.

The problem of finding the optimum stand condition may be expressed as: find the initial stand condition X_0 which gives rise to the stand X_t in t years time, such that the increase $X_t - X_0$ is maximized. The initial state X_0 may be zero in the case of an even-aged stand which is ultimately clearfelled. Meyer (1952) expressed this as the stand structure in which ''current growth can be removed periodically while maintaining the diameter distribution and the initial volume of the forest''. Many simple optimization studies rely on the existence of this ''sustainable distribution'' (Adams and Ek 1974, 1975):

$$N_{d,t+1} \geq N_{d,t} \text{ for all } d.$$

where $N_{d,t}$ is the number of trees in a diameter class d at time t. This ensures that at any future time, a number of trees can be removed from each class to return to the original distribution. Michie (1985) used a matrix model to develop a more general case which converts any initial stand structure to an investment-efficient sustainable distribution in a pre-determined number of harvests.

Adams and Ek (1974) maximized the stumpage value of stems removed to restore the initial diameter distribution with the constraints that the number of stems removed from any class must always be non-negative, the specified initial basal area must be maintained, and that the number of trees in each diameter class must never be less than zero. This simple analysis led to a considerable increase in predicted value of cut, of both stumpage value and in volume, over the existing management guides. They considered five and ten year cutting cycles in their analysis, but made no attempt to consider a variable cutting cycle in their study. It is possible to automatically determine both the stand condition (number of trees in each class) and the cutting cycle that will lead to the optimum benefit. The problem of the optimal conversion of some stand to the optimum stand condition was formulated so as to maximize the revenue from a specified number of cuts which result in the specified optimum stand condition after the final cut. They gave an example where a stand could have been converted immediately to the optimal condition, but it would have been uneconomic to do so. Their analysis led to an optimal conversion schedule which maximized the perceived benefits. Adams and Ek (1974) did not consider the holding cost of the residual growing stock, so their analysis yielded different results to those obtained from dynamic present net value studies (e.g. Haight 1985).

Bare and Opalach (1987, 1988) assumed that the sustainable diameter distribution in spruce-fir forests could be modelled by a Weibull distribution. This assumption enabled computational efficiencies, but such a uni-modal distribution may be inappropriate for TMF. They found that the ''investment efficient'' sustainable equilibrium diameter distribution associated with maximum land expectation value differs from the maximum managed forest value; both differ dramatically from the maximum board foot volume growth. These and other studies demonstrate the interrelationship between management objectives and optimal stand structures.

These and many other optimization studies used a simplified growth model to provide a tractable model for their analyses, and such simplification may influence the results. Haight and Monserud (1990a,b, Monserud and Haight 1990) demonstrated a method for optimizing any-aged management of mixed species stands using a tree list growth model (Prognosis) without modification. They did not assume a sustainable diameter distribution, but allowed any silviculture ranging from selection logging and shelterwood systems

to clearfelling with natural regeneration or planting. Their formulation of the problem was to define the n-dimensional vector $\mathbf{x}(t)$ as the initial state (i.e. the present stand table) at the beginning of time period t, and let $\mathbf{u}(t)$ be an m-dimensional vector of controls where n and m are the number of state and control variables respectively. Typically, $\mathbf{x}(t)$ represents the distribution of trees by diameter and species classes, and $\mathbf{u}(t)$ represents the harvest levels for various classes. Each element $u_j(t),\ j = 2, \cdots, m$, of $\mathbf{u}(t)$ is defined as the proportion of the trees harvested from diameter and species class j and is constrained between 0.0 and 1.0. The smallest size class $u_1(t)$ is unconstrained to allow for planting, and when $u_1(t) < 0$ it represents the number of seedlings planted, and when $u_1(t) > 0$ it represents the number of seedlings thinned. Let $R[\mathbf{x}(t), \mathbf{u}(t)]$ denote the revenue obtained in period t, where the resource is in state $\mathbf{x}(t)$ at the beginning of this period before harvesting, and the harvest and planting control $\mathbf{u}(t)$ takes place at the beginning of the period. Then letting δ denote a discount factor ($\delta = 1/(1 + r)$, where r is discount rate), and T the planning horizon, the optimization problem can be expressed as

$$\max_{\{\mathbf{u}(t), t=0,1,\cdots,T-1\}} J_T[\mathbf{x}(0)] = \sum_{t=0}^{T-1} \delta^t R[\mathbf{x}(t), \mathbf{u}(t)] \ + \ \delta^T G[\mathbf{x}(T)]$$

where $\mathbf{x}(0)$ is the initial state and $G[\mathbf{x}(T)]$ is the terminal payoff function for the stand in state $\mathbf{x}(T)$. The maximization was solved subject to an n-dimensional difference equation for the stand dynamics which implicitly represents natural regeneration, tree growth and survival:

$$\mathbf{x}(t+1) = f[\mathbf{x}(t), \mathbf{u}(t)], \quad t = 0, 1, \ldots, T-1$$

The solution to these equations is the control variable set $\{\mathbf{u}(t), t = 0, 1, \cdots, T-1\}$ that maximizes $J_T[\mathbf{x}(0)]$, the present value of the existing stand over the T-year planning horizon. To solve these equations, Haight and Monserud (1990a,b) used a coordinate search process (Hooke and Jeeves 1961, Bazaraa and Shetty 1979) which was modified for their application. They found that even-aged plantation management and uneven-aged shelterwood systems were both capable of producing the same high level of yield indefinitely.

Haight and Monserud (1990a) cautioned that optimal solutions to any-aged management problems may vary considerably depending on initial estimates for parameters to be estimated. Wide diameter classes and broad species groups provided more robust results. They stressed that sensitivity analyses were essential to investigate the sensitivity to starting condition, and to ensure that a global rather than a local optimum had been reached. Optima may also be sensitive to the planning horizon, and long term simulations (e.g. 120 years) may be necessary for uneven-aged forests.

Information concerning the sensitivity of the optimum is as important to the manager as the optimum itself. In many cases it may be difficult or even impossible to achieve the optimum, and the manager will want to know how close to the optimum stand condition his forest needs to be to achieve a near optimal return. The optimum may well be attained over a wide range of stand conditions, and sensitivity analyses should be carried out to indicate, for example, the range of stand conditions which return a yield within five percent of the maximum. The forest manager may then strive to attain a stand condition somewhere within this range, and may well exert his own beliefs, based on personal judgement and experience, as to the best condition for stands under his control.

12.4 Yield Prediction

Yield prediction is probably the most important application of many growth models. A number of factors need to be considered, to ensure that yield estimates are not biased. Yield estimates prepared by multiplying the productive area of the forest estate by the theoretical per-hectare sustainable yield usually overestimates attainable yields. More detailed simulation studies are required to provide reliable estimates of sustainable yields for a forest estate.

Determining the yield from a single stand with the aid of a growth model is a relatively simple matter, but extending the concept to the whole forest estate requires some account to be taken of the distribution of yield over time. The manager may strive for a ‘‘non-declining even flow’’ of timber onto the market (Gunnersen 1981), even though the market may be capricious and the forest estate far removed from the ‘‘normal forest’’. This generally requires that some management units may need to be cut before, after, heavier or lighter than the optimal cutting rules would suggest.

Managers may also wish to capitalize on buoyant market opportunities by temporarily increasing the harvest. There is no problem with this, as the sustained yield is only a guide, and variations in cutting cycle length and intensity are inevitable. Haight (1990, Haight and Smith 1991) investigated thinning strategies in situations where prices vary stochastically. When the sustained yield is temporarily exceeded, the standing crop and its increment is reduced, and production and future yields may be depressed for some time. Similarly, if the stand becomes overstocked through ‘‘undercutting’’, production may also be depressed, although the standing volume will continue to accumulate to the maximum for the site.

Simulation studies can assist the manager to find the best path through the sometimes contradictory requirements of maximizing sustained yield and achieving a non-declining even flow. Such studies require basic resource data (i.e. stand table and nett area) for each management unit within the forest estate and a good growth model.

12.4.1 Averaging Yields from Individual Plots

Hann (1980) observed that yields estimated from individual one hectare plots were more accurate than an estimate from the mean of these plots. This leads to the question: should the plots be projected individually, and the projections averaged, or should the easier approach of projecting the average of all the plots be adopted? The potential for bias in projecting the averages is evident. Suppose that the forest under consideration is overstocked on about half its area, and understocked elsewhere. Then the average of the plots will indicate a near optimal stocking, and will forecast yields higher than the average of the individual projections. However, if the forest is stratified into a number of units, each homogeneous with respect to site and stand composition, it should be possible to project the plot averages for any stratum. Hann (1980) observed a loss of accuracy in projecting the average of 32 immediately adjacent plots, but considered this loss to be acceptable for most applications. Hagglund (1981) also observed that for uniform, even-aged stands, the projection of the stand mean resulted in the same estimate as the average of the projection of each individual plot. However, Hagglund stressed that these results apply only to homogeneous and well managed stands using objective and accurate methods of inventory, and need not apply under other circumstances.

Moeur and Ek (1981) compared predictions from individual plots, averages for homogeneous stands, and averages for forest types. Their study involved 134 permanent plots each 0.058 hectares, established according to a systematic design with random starts. The homogeneous stands were defined from inspection of 1:16 000 scale aerial photographs, and forest types were classified according to the major overstorey species, as aspen, red pine or jack pine. Although all their projections overestimated the actual yield, the best predictions were obtained from the average of the individual plot predictions, and the greatest overestimates resulted from projecting the average for the forest type. These findings are consistent with studies by Smith and Burkhart (1984) and McKay (1990).

In view of the expense involved in developing a growth model and obtaining resource data, it seems unwise to economize on computing costs by projecting plot averages rather than individual plots. If time and computing resources are limited, the average of homogeneous strata may be projected, but it is preferable to project the individual plots and to determine the average yield after the projections.

12.4.2 Cutting Cycle Analysis

Cutting cycle analysis has been extensively used for yield forecasting in uneven-aged forests, partly because of its ease of application and because it is the most reliable technique which can be performed without computers. The basic method (McGrath and Carron 1966, Davis and Johnson 1987:48) is to nominate a cutting cycle length, construct a typical stand table, project this stand to the mid-point of the cutting cycle, and apply a logging rule to determine the loggable volume. As the actual time of logging is not known, the mid-point of the cutting cycle is used as a compromise. The annual yield is determined by dividing the loggable volume by the cutting cycle length.

Cutting cycle analysis has a number of advantages over approaches such as area control and growth tables. As the method employs the current stand table and a logging rule which ideally approximates current field practice, it is able to indicate the loggable yield. Continuing the analysis for two or three cycles indicates

the long term yield, and the viability of the nominal cutting cycle length and logging rule.

The method poses a number of questions which require subjective decisions and which may have a substantial impact on forecasts: What if the yields derived from successive cycles differ? Should the yield be set at the average of these, should the nominal cutting cycle length be altered, or should the logging rule be changed? Is the ‘‘typical’’ stand employed representative? The method can be improved by stratification on the basis of site productivity and standing volume, and by simulating individual plots rather than stratum averages. Other deficiencies include the assumption of a fixed cutting cycle for all stands in the stratum, the assumption of harvesting at mid-cycle, and the implicit assumption that all stands will be cut in the same sequence in subsequent cutting cycles (Phillis 1971).

12.4.3 Yield Scheduling

Yield scheduling by heuristic (trial and error) simulation can overcome many of the deficiencies of traditional cutting cycle analysis. It attempts to emulate the sequence of harvesting across the resource, and determines the most appropriate ‘‘cutting cycle’’ for each individual plot, rather than using the nominal cycle in cutting cycle analysis. Yield scheduling enables the user to specify any allowable cut and examine the outcome. In this way, it reveals for how long a given harvest can be sustained. Heuristic simulation does not provide the maximum sustainable yield, but takes an initial estimate by the user and provides information to enable the user to make a better estimate for a subsequent iteration (Davis and Johnson 1987:656, Leuschner 1990:164).

Grosenbaugh (1955) recognized the deficiencies in the horizontal cut and cutting cycle analysis approaches, and advocated the recognition of homogeneous ‘‘record-units’’ which were to be the sole area unit for all mapping, sampling, forecasting and operational work. He also insisted that yield forecasting should recognize the actual order of working over the resource. Phillis (1971:239) reported a trial of Grosenbaugh’s ‘‘diagnostic survey technique’’ in an irregular eucalypt forest, and reported that it was efficient in producing detailed yield estimates and other information useful for operational management at a cost comparable to the established continuous forest inventory system.

Vanclay and Preston (1989) gave an example of heuristic simulation to provide yield schedules for Queensland TMF. Their resource was partitioned into management units which formed the basis for management and prediction, and which were further stratified into homogeneous subunits for efficient sampling. Their simulation system enabled several constraints to be specified, and included the specification of minimum yields per hectare, species mixes and other criteria, and this ensured that the predicted harvesting schedule made sense from a silvicultural and management viewpoint. Their study illustrated discrepancies between cutting cycle analysis and yield scheduling approaches to predicting sustainable yields, and demonstrated the simulation of a sustained harvest from the TMF in their study area.

Heuristic simulation offers some advantages over the widely-used alternatives of linear programming and other mathematical programming techniques. It is conceptually very simple, and easy for forest managers to comprehend. It avoids many of the problems in determining a suitable objective function (Haight 1987) and end condition (Davis and Johnson 1987:614, Haight 1988). Rose (1984) discussed the implications of aggregation needed to formulate mathematical programming solutions and their sensitivity to basic assumptions, and suggested that heuristic approaches may be a more appropriate approach. Johnson and Tedder (1983, Davis and Johnson 1987:668) discussed several advantages of heuristic simulation over linear programming, including the ability to portray inventory in greater detail, shift areas in and out of production more easily, to produce analyses at lower cost, and to find feasible solutions more easily.

12.4.4 Linear Programming

Linear programming and other mathematical programming techniques have been very useful and widely used for estimating and regulating timber harvests from plantations (e.g. Clutter *et al.* 1983:272, Davis and Johnson 1987:592, Leuschner 1990:82), but have been used to a lesser extent in natural forests. Basically, linear programming (LP) will sort through a list of choices and select one that satisfies the specified constraints and maximizes the objective (e.g. present net value). Thus an LP model requires three components: a set of decision variables, a set of constraints, and an objective function. In addition, it

requires a matrix generator to formulate the list of choices, a LP package to solve the matrix, and a report writer to summarize the results in an intelligible form. Davis and Johnson (1987:592) give a good introduction to the topic. Johnson and Tedder (1983, Davis and Johnson 1987:668) described several advantages of linear programming, including simultaneous examination of several alternatives, the ability to portray unusual yield trajectories and to accommodate more constraints, and the ability to find the optimum.

FORPLAN is probably the LP model most widely used in forestry, and the U.S. Forest Service has a legal obligation to use it in forest management planning (Mitchell and Kent 1987, Leuschner 1990:204). Yet despite this prominence in the field, it is complex, and is not well documented or used (Johnson 1987, Kent *et al.* 1988). Perhaps the major criticism of FORPLAN is that it has become an end in itself, rather than a means to investigate better forest management (Sedjo 1987). FORPLAN does more than timber harvest scheduling, and its major uses are in multiple use planning and in resolving conflict (e.g. Mealey 1987, McKenney 1990). Whilst FORPLAN and other mathematical programming systems are undoubtedly useful, they do not provide an easy option, and considerable skill and resource information are necessary to use these techniques.

12.4.5 Precision and Sensitivity

It is difficult to estimate the likely accuracy of a yield prediction. As Leary *et al.* (1979a) pointed out, there are two sources of error involved: error in assessing the initial state, and error in the growth prediction for the plot. The former is a problem of resource inventory, and will not be discussed here. It is significant however, and may contribute the majority of error associated with predictions (Mowrer and Frayer 1986, Mowrer 1989).

Ek and Dudek (1980) and Reynolds (1984) give formulae for estimating approximate standard errors associated with growth projections for both single plots, and for plot means. Ek and Dudek (1980) found that typical standard errors for growth estimates were about 15 to 45 percent for projections of 15 to 30 years over a broad range of stand conditions in temperate natural forests. Estimates for plantations are usually better, whilst it is likely that estimates for TMF will be less precise until better data are obtained.

The need for sensitivity analyses in optimization studies has already been discussed. However, it is equally important in yield studies to carry out sensitivity analyses to determine the consequences on yield estimates of bias in the resource inventory. It is possible that the plots used to derive the yield estimates were not representative of the forest, and it is important for the manager to know how sensitive the yield estimates are to such inaccuracies in the resource inventory. Gertner and Dzialowy (1984) found that accurate site productivity estimates were particularly important.

12.5 Risk Analyses with Stochastic Models

Stochastic models initially pose three questions. How many simulation runs are required in order to give a good indication of the most likely outcome and its distribution? What is the best measure of the most likely outcome - the mean or the median? How should the confidence limits be estimated?

The number of simulations used to derive results published in the literature varies greatly. Ek and Monserud (1979) used four runs, Weber *et al.* (1986) used ten, Shifley *et al.* (1982) used fifty, and Botkin *et al.* (1972) used one hundred to gain estimates of the mean result. Gertner (1987a) needed 5000 replications to get stable estimates of variance. These studies do not indicate the basis for the selection of these numbers. The appropriate methods to use to estimate the number of simulations required, the most likely outcome, and the confidence limits depends largely upon the distribution of the yield estimates given by the model.

If the distribution of yield predictions from the model is normal, the common formulae may be used to calculate the mean, the variance and the required number of simulations. Ek and Monserud (1979) found that output from their model was normally distributed. However, if the distribution of predictions is not normal, non-parametric estimates may be necessary to determine the most likely outcome and its variability.

In determining whether the distribution is likely to be normal, two approaches may be used. If, during the verification and validation process, rigorous testing reveals that the distribution is always close to normal, it may be reasonable to assume that a normal distribution will generally result, and that the conventional statistics may be used. An alternative is to test for normality within the computer program during the execution of a series of simulations. This latter approach is likely to be unnecessarily intensive of computing resources, and it may be more efficient to assume a non-normal distribution in all cases. It is better to err on the conservative side, and assume that the distribution is not normal, particularly where one-sided long-tailed distributions may arise.

If the distribution is significantly different from the normal distribution, other approaches need to be used to determine the number of simulations, the most likely outcome and its variance. The median provides an unbiased estimate of the expected outcome, and is given by the $(n+1)/2$th observation of the sample, where n is an odd number and samples are ranked by expected outcome. Do say nine runs, then keep adding runs (using a power function e.g. $2^n + 1$) until no change in the median is observed. For a more precise result, keep adding more runs until the difference between the two observations either side of the median is small (i.e. $Y_{(n+3)/2} - Y_{(n-1)/2} < \delta$, where n is odd and Y_i are ranked in increasing order).

The variance of the population may be estimated as quarter of the range, but this test is not robust and performs poorly for non-normal distributions with long tails (Snedecor and Cochran 1980:137). A better approach may be to observe the confidence interval about the predictions directly from the simulated outcomes rather than from some formula. If, for example, the ninety percent confidence limits are required, the largest and smallest observations in 19 samples will give a rough estimate, the second largest and second smallest of 39 samples a better indication, as will the fifth largest and fifth smallest of 99 samples. For any continuous distribution, the probability that all of the next k observations are between the smallest and largest values in the current sample of size n is (Hahn and Nelson 1973, Reynolds 1984):

$$1 - \alpha = \frac{n(n-1)}{(n+k)(n+k-1)}$$

The number of samples required may be dictated by two requirements: the need to gain a reasonable estimate of confidence limits for yield, and the need to estimate the median prediction with some precision. If reasonable estimates of confidence limits on the predicted yield are required, 30 or more simulations may be dictated. However, if concern is for the median, another approach may be used. Snedecor and Cochran (1980:137) give a formula which gives conservative confidence limits for the population median, and is valid for any continuous distribution:

$$i, j = \frac{(n+1) \pm z\sqrt{n}}{2}$$

where z is the normal deviate corresponding to the desired confidence probability, and the ith and jth observations indicate the conservative confidence limits about the median. Other nonparametric tests are given in standard texts (e.g. Lehmann 1975, Hettmansperger 1984).

12.6 Conclusion

Nobody can guess, at the time of model construction, all the possible uses to which a growth model may be put. Some of the immediate uses have been discussed, and these are likely to prove of considerable importance and value in forest management.

It is stressed that a growth model must not remain a sophisticated complexity, alien to the forest manager, but must be made available for use on a regular basis as an every-day tool for better forest management. In short, that means that the growth model should be easy to use, well documented and readily available.

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